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THE INFLUENCE OF PORE-WATER TENSION ON THE STRENGTH OF CLAY

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[Plate 17]

CONTENTS

Substantial pore-water tensions are set up by the removal, under undrained conditions, of the confining pressure from saturated clay samples from which the pore water has been free to drain during consolidation. These pore-water tensions control the mechanical behaviour of the unconfined samples, and in particular their strength and brittleness.

In the present paper the magnitude of the pore-water tensions and the change in strength on stress release are estimated theoretically for the range of pressures within which the sample remains fully saturated. Tests on samples of two clays consolidated in the triaxial apparatus under a wide range of pressure indicate that the limiting pore-water tension above which the sample ceases to remain fully saturated is related to the equivalent pore diameter. Above this limit the loss in strength on stress release is very marked.

The experimental results also show the dramatic change in brittleness resulting from high pore-water tensions. A large reduction in total stress without a change in water content is sufficient to change the failure mechanism from a plastic failure with some strain-softening to a brittle failure in which the sample shatters.

The significance of the results in relation to sampling and testing of soils for engineering purposes is briefly considered.

1. Introduction

When a pressure is applied to a saturated porous material the load is carried wholly by the solid skeleton only if the pore fluid is free to flow out of the voids. If this is prevented by the rapidity of the loading, by the low permeability of the material or by the length of the drainage path, then the load is shared between the solid skeleton and the fluid in the pore space in a ratio which is a function of their compressibilities.

An expression for the ratio of the change in pore pressure Δu to the change in the total normal stress $\Delta\sigma$ was derived by Bishop & Eldin (1950):

$$\frac{\Delta u}{\Delta \sigma} = \frac{1}{1+n(C_\mathrm{w}/C)}, \tag{1}$$

where n denotes the porosity, C_w the compressibility of the pore fluid and C denotes the compressibility of an element of the porous material with respect to a change in ambient stress accompanied by zero change in pore pressure.

In the derivation of this expression it was assumed that the compressibility C_s of the solid material forming the skeleton of the soil or other porous material might be neglected. A more general expression which includes this term has been given by Bishop (1966, 1973)

$$\frac{\Delta u}{\Delta \sigma} = \frac{1}{1+n[(C_\mathrm{w}-C_\mathrm{s})/(C-C_\mathrm{s})]}. \tag{2}$$

However, for a soil saturated with water, $C_\mathrm{w} = 49 \times 10^{-5}\,\mathrm{m^2/MN}$ and $C_\mathrm{s} = 2-3 \times 10^{-5}\,\mathrm{m^2/MN}$. The value of the compressibility of the solid skeleton is generally larger by several orders of magnitude, being equal to $3000 \times 10^{-5}\,\mathrm{m^2/MN}$ for a typical sample of heavily overconsolidated London Clay, for example. In these circumstances† the expression is dominated by the ratio of C to C_w and, since C is large compared with C_w, the ratio $\Delta u/\Delta\sigma$ approximates to unity in the low and medium stress range (being 0.994 for the example quoted above).

This conclusion is supported by direct measurements of pore pressure change in saturated

† For rocks and soils under high confining pressures the very low values of the compressibility C make the C_s term of relatively greater importance, and lead to low values of the ratio $\Delta u/\Delta\sigma$. For a value of $C = 9 \times 10^{-5}\,\mathrm{m^2/MN}$ representative of sandstone under a high confining pressure, $\Delta u/\Delta\sigma = 0.47$.

sand (Bishop & Eldin 1950) and in saturated clay (Taylor 1944; Bishop 1960). Indeed, the most reliable way of determining whether or not a soil sample is fully saturated is to measure the ratio $\Delta u/\Delta\sigma$ under undrained conditions.

Although the value of the compressibility C for a normally consolidated clay soil is very approximately proportional to the reciprocal of consolidation pressure it is only at relatively high consolidation pressures that the ratio $\Delta u/\Delta\sigma$ departs significantly from unity. From the observed compressibility of the London Clay samples discussed in later sections of this paper the calculated value of $\Delta u/\Delta\sigma$ for an increase in stress would be 0.97 and 0.80 for consolidation pressures of 20.7 MN m^{-2} (3000 lbf in^{-2}) and 62.1 MN m^{-2} (9000 lbf in^{-2}) respectively. As the soil structure is not elastic the rebound modulus is substantially different from that observed on first loading, and estimated values of $\Delta u/\Delta\sigma$ for unloading drop to 0.94 and 0.63–0.77 respectively.†

Now consider an element of saturated soil which has been subjected to a total normal stress equal in all directions of σ, the pore water being permitted to drain freely to give an initial value of the pore pressure equal to u. The effective stress, which controls the volume change and strength characteristics, is given to a very close approximation, in the case of saturated soils (Terzaghi 1936; Bishop & Eldin 1950; Bishop 1959; Skempton 1960; Bishop & Blight 1963; Bishop 1966) by the expression

$$\sigma' = \sigma - ku, \tag{3}$$

where k is a parameter which differs only marginally from unity in the case of uncemented soils at low consolidation pressures.

In principle the value of k depends on whether the effective stress equation is used with reference to changes in volume or to changes in shear strength. In the former case an expression was obtained by Bishop (1953):

$$k_v = (1 - C_s/C). \tag{4}$$

For changes in shear strength Skempton (1960) derived the expression:

$$k_s = [1 - a(\tan\psi/\tan\phi')], \tag{5}$$

where a denotes area of contact between the particles, per unit gross area of material; ψ denotes the angle of intrinsic friction of the solid material of the soil grains, and ϕ' denotes the angle of shearing resistance of the granular material. For the values of C for London Clay used in the calculation of the ratio $\Delta u/\Delta\sigma$, values of k_v of 0.999, 0.994 and 0.939 are obtained for the over-consolidated specimen, and the specimens consolidated at pressures of 20.7 MN m^{-2} (3000 lbf in^{-2}) and 62.1 MN m^{-2} (9000 lbf in^{-2}) respectively. The value of k_v is stress-path dependent, and for unloading the last two values drop to 0.989 and 0.887 respectively.

The value of k_s is less readily estimated due to uncertainty about the value of a in particular, especially for clays. A very accurate experimental examination in the low effective stress range ($\sigma - u$ = 0.36 MN m^{-2} (52.6 lbf in^{-2})) using pore pressure changes of up to 27.6 MN m^{-2} (4000 lbf in^{-2}) indicated that in the case of sand k_s did not differ from unity by more than 5×10^{-5}, which was the limit of accuracy of the test (Bishop 1966; Skinner 1975). If we assume that, for small values, a increases linearly with consolidation pressure (Skempton 1960), it follows from

† The data from which these estimates are made is given in table 4. Few direct observations are currently available of $\Delta u/\Delta\sigma$ for porous materials of low compressibility, where the departure from the truly 'undrained' condition due to the flow of water into the measuring system becomes of particular importance. Further reference is made to this problem by Bishop (1973) and examples of system compressibility are given by Wissa (1969) and Bruhn (1972).

equation (5) that the difference between the actual value of k_s and unity increases linearly with consolidation pressure. It may then be inferred that this difference would not exceed 2.8×10^{-3} at 20.7 MN m^{-2} (3000 lbf in^{-2}) and 8.6×10^{-3} at 62.1 MN m^{-2} (9000 lbf in^{-2}). This would indicate values of k_s not lower than 0.997 and 0.991 respectively. There is no evidence † that the values for clay would be lower, though clearly the physical nature of the inter-particle contact will be significantly different.

If the element of soil under consideration is now subjected to a change in total normal stress $\Delta\sigma$ under conditions of zero drainage, the pore pressure will change by Δu. If the ratio $\Delta u/\Delta\sigma$ is denoted by the parameter B (Skempton 1954) the change in effective stress is given by the expression:

$$\begin{aligned}\Delta\sigma' &= \Delta\sigma - k\,\Delta u\\ &= \Delta\sigma - \Delta u + (1-k)\,\Delta u\\ &= (1-B)\,\Delta\sigma + (1-k)\,B\,\Delta\sigma.\end{aligned} \tag{6}$$

In the low and medium range of consolidation pressures the value of B will differ from unity by less than 1 % and k is likely to differ from unity by an even smaller amount, probably less than 0.1 %. Hence the change in effective stress, and thus in volume change and strength, will lie close to the limits of direct measurement in the laboratory. This is confirmed by the negligible increase in undrained strength with increase in total normal stress reported from tests on saturated clays (Terzaghi 1932 and 1936; Jurgenson 1934; Golder & Skempton 1948; Bishop & Bjerrum 1960) and from tests on saturated sand (Bishop & Eldin 1950). Two important exceptions have, however, been reported:

(*a*) *Strongly dilatant sands tested at low confining pressures* (*Bishop & Eldin* 1950)

Here the application of the principal stress difference producing failure was associated with a substantial decrease in pore pressure. In triaxial tests at low cell pressures this decrease resulted in negative pore pressures (gauge) and the stress–strain curve terminated prematurely, due to the inability of the water in the pore space and in the measuring system to withstand the tension necessary to maintain full saturation.

(*b*) *Unconfined compression tests on some clay samples* (*Bishop* 1947; *Golder & Skempton* 1948; *Bishop & Henkel*, 1962)

It has been observed that the unconfined compression strength is often lower than the average of the strengths at higher confining pressures, although the moisture content is the same and no drainage has been permitted during shear. Since the samples are nominally saturated this difference has usually been attributed to the presence of fissures (*loc. cit*). However, it may be noted that throughout an unconfined compression test the pore-water pressure is negative (see, for example, Bishop 1960), and from the high strength of some samples substantial pore-water tensions may be inferred, though they cannot be measured directly (see, for example, Croney & Coleman 1960). In this respect clays appear to differ from the sand samples referred to in the preceding paragraph.

It is therefore of interest to examine the influence of high pore-water tensions on the strength of clay tested at constant water content in the absence of fissures.

† A series of similar tests on clay (Kumapley 1969) are consistent with tests on sand, but are not capable of the same accuracy due to the lower pressures used and to the problems of pore pressure equilization in soils of low permeability.

2. Prediction of pore-water tension and change in strength on stress release

Figure 1 illustrates the predicted pore pressure in an undrained saturated sample which has initially been consolidated under a total normal stress σ_0 with free drainage of the pore water to atmospheric pressure and then subjected to a change in total normal stress under undrained conditions. The relation between total normal stress σ and pore pressure u (gauge) is given by the expression

$$u = B(\sigma - \sigma_0). \tag{7}$$

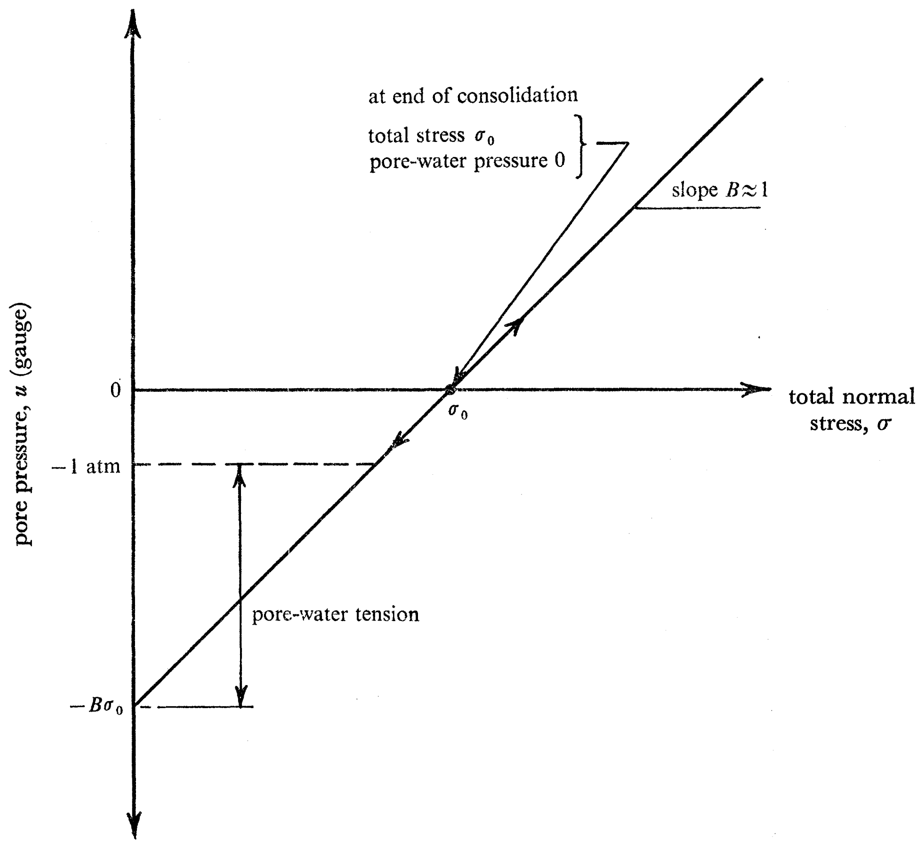

Figure 1. Relation between pore-water pressure and total normal stress for an undrained change in stress after consolidation with the total stress equal to σ_0 and the pore water free to drain to atmosphere. (Full saturation assumed.)

For an unconfined specimen the initial pore pressure before shear would thus be equal to $-B\sigma_0$, and the amount by which this negative pore pressure falls below minus one atmosphere will represent tension in the pore water. If B approximates to unity under these conditions, then the pore water of samples consolidated at a total normal stress in excess of plus one atmosphere should be in a state of tension when the sample is unconfined. In the more general case (figure 2) in which the sample drains, not to atmospheric pressure, but against a back pressure u_0, the relation between total normal stress and pore pressure is given by the expression

$$u = u_0 + B(\sigma - \sigma_0). \tag{8}$$

In the present series of tests values of σ_0 of up to 62.1 MN m^{-2} (9000 lbf in^{-2}) have been used, associated with $u_0 = 0$. A theoretical maximum pore-water tension of $B \times 62.1$ MN m^{-2} could therefore have been induced in the pore water.* Taking the value of B for unloading at this consolidation pressure as 0.63 we have a tension of 39.1 MN m^{-2} (5670 lbf in^{-2}).

As will be seen from the test results, a sample of London Clay could not sustain this pore-water tension and remain saturated.

To this negative pore pressure must be added the change in pore pressure resulting from the application of the stress difference necessary to cause undrained failure. In contrast to the case of dilatant sands, this additional pore pressure change is positive in most clays unless they have been heavily over-consolidated before the undrained test is carried out.

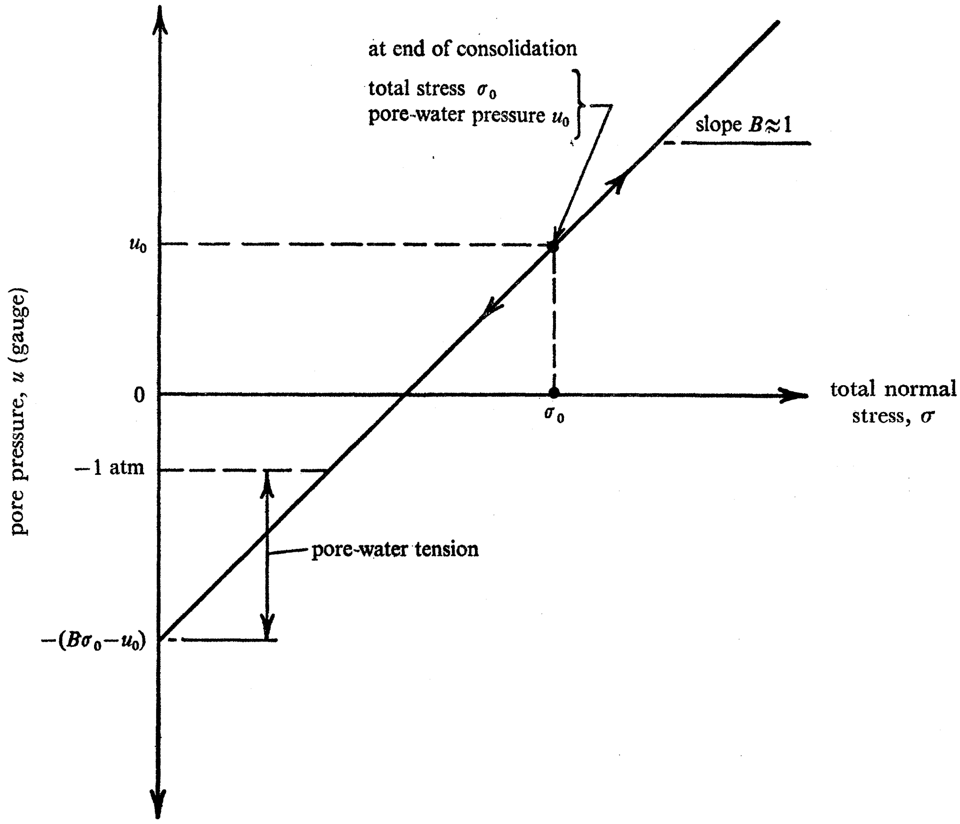

FIGURE 2. Relation between pore-water pressure and total normal stress for an undrained change in stress after consolidation with the total stress equal to σ_0 and the pore pressure equal to u_0. (Full saturation assumed.)

This additional change in pore pressure may be conveniently expressed in terms of the more general expression (Skempton 1954):

$$u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)], \tag{9}$$

where $\Delta\sigma_1$ denotes the change in major principal stress (i.e. the axial stress in the triaxial compression test), $\Delta\sigma_3$ the change in minor principal stress (in the conventional triaxial compression test $\Delta\sigma_3 = \Delta\sigma_2$ and equals the change in cell pressure), and A a pore pressure parameter which varies with clay type and stress history. For normally consolidated clays A is positive and generally close to unity at failure if the consolidation has been carried out under isotropic conditions (see, for example, Bishop & Henkel 1962, Table 5). For over-consolidated clays, which have been consolidated under a maximum effective consolidation pressure p_c and allowed to swell under a

* Neglecting the 0.1 MN m^{-2} difference between gauge and absolute pressure.

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smaller pressure p before the undrained test stage, the value of A_f at failure becomes negative only where the over-consolidation ratio p_c/p exceeds 4 or 5 in typical cases of remoulded clay (Bishop & Henkel 1962, Fig. 82). Undisturbed samples of London Clay which have been heavily over-consolidated in nature in general do not show a negative A value at the point of maximum stress difference (Bishop, Webb & Lewin 1965; Agarwal 1967), though in undisturbed Weald Clay a value as low as -0.62 has been observed (Bishop & Henkel 1962).

The pore pressure at failure is obtained from equations (8) and (9) as

$$u = u_0 + B(\sigma - \sigma_0) + BA_f(\Delta\sigma_1 - \Delta\sigma_3)_f. \tag{10}$$

The maximum shear stress at failure under undrained conditions may be denoted c_u and is equal to $\frac{1}{2}(\Delta\sigma_1 - \Delta\sigma_3)_f$. The ratio c_u/p, where p is the effective consolidation pressure, closely approximates to a constant for normally consolidated samples of a given clay. Equation (10) may therefore be written

$$u = u_0 + B(\sigma - \sigma_0) + BA_f 2(c_u/p)\,(\sigma_0 - u_0). \tag{11}$$

For the present series of tests on London Clay, $c_u/p = 0.20$ for normally consolidated samples and $A_f = 1.25$ to a close approximation.† Taking the case where $u_0 = 0$ and putting $B = 1$, we have: initial pore pressure in the unconfined state

$$u = -\sigma_0, \tag{12}$$

pore pressure at failure in the unconfined state

$$\begin{aligned} u &= -\sigma_0 + 1 \times 1.25 \times 2 \times 0.20\sigma_0 \\ &= -0.50\sigma_0. \end{aligned} \tag{13}$$

The pore-water tension will thus be highest in the unconfined state before the axial load is applied and will decrease very substantially before failure.‡

Where B or k differ significantly from unity, it is apparent from equation (6) that a reduction in confining pressure will result in a decrease in effective stress. The unconfined sample will therefore behave as a slightly overconsolidated material, but the over-consolidation ratio p_c/p will be relatively small (it is equal to $1/B$ if $k = 1$) and change in the value of A_f will therefore not alter the general conclusion of the previous paragraph.

However, this decrease in effective stress will be reflected in a reduction in the undrained strength ($-\Delta c_u$) of the sample. As will be seen from the test data reported in this paper the relation between the undrained strength c_u and the consolidation pressure p is almost linear on first loading from a slurry. However, this relation is not fully reversible (Figure 3). The results of the tests on London Clay (series 2) reported in this paper indicate that on the reduction of the effective consolidation pressure before undrained shear we have

$$\begin{aligned} [\Delta c_u/\Delta p]_{\text{unloading}} &= m(c_u/p)_{\text{first loading}} \\ &= mn, \end{aligned} \tag{14}$$

† From the results of undrained tests with pore pressure measurement at relatively high consolidation pressures reported by Bishop *et al.* (1965).

‡ This is in contrast to the position for dilatant sands, for which A_f may drop as low as -0.42, as in the tests reported by Bishop & Eldin (1950). For cohesionless soils where the angle of internal friction is ϕ', the theoretical lower limit of A_f is given by the expression (Bishop 1952):

$$A_f = -(1 - \sin\phi')/2\sin\phi'.$$

where $m = 0.5$ for London Clay consolidated from a slurry, for $-\Delta p/p_{\max} = \frac{1}{2}$, and n is a constant approximating to 0.2 for this batch of London Clay. Taking $k_s = 1$, we have, from equation (6)

$$\Delta p = \Delta\sigma' = (1-B)\,\Delta\sigma. \tag{15}$$

The reduction in strength consequent on a reduction in total stress $-\Delta\sigma$ is thus

$$-\Delta c_u = (1-B)\,(-\Delta\sigma)\,mn. \tag{16}$$

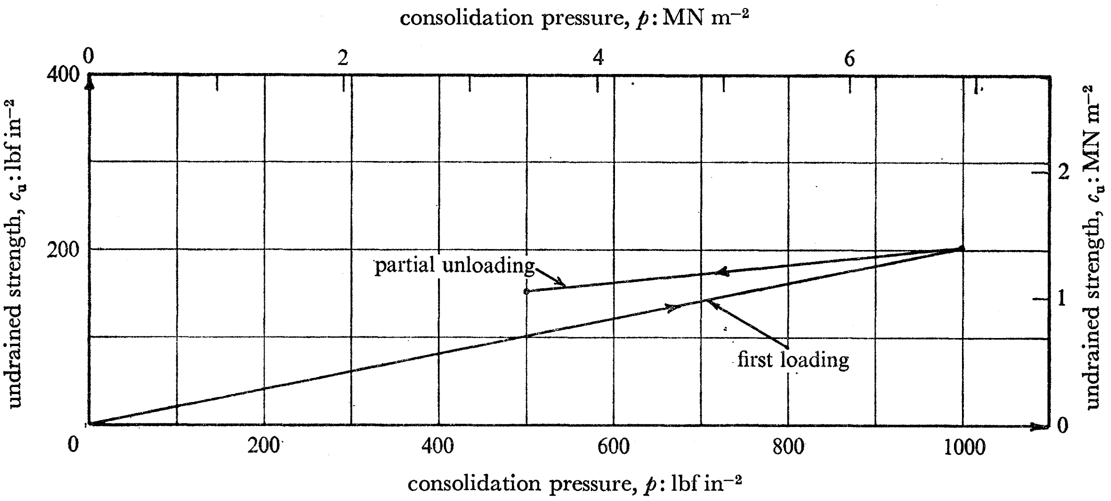

FIGURE 3. The relation between undrained strength c_u and consolidation pressure p on first loading and on partial unloading. Data from tests on London Clay, series 2.

Before the reduction in total stress, the effective consolidation pressure is $\sigma_0 - u_0$ and the undrained strength $(c_u)_0$ is equal to $n(\sigma_0 - u_0)$. The proportional reduction in strength r is

$$\frac{-\Delta c_u}{(c_u)_0}$$

and is thus given by the expression

$$r = -\frac{m(1-B)\,\Delta\sigma}{\sigma_0 - u_0}. \tag{17}$$

The accuracy of predictions made on the basis of this expression is subject to two qualifications. The stress decrement ratio $-\Delta p/p_{\max}$ resulting from an undrained reduction in total stress is smaller than the values of this ratio used in obtaining representative values of m. The value of $m = 0.5$ obtained for this clay and from earlier tests on London Clay by Donald (1961) may therefore represent a slight overestimate.

Similarly the value of expansibility used in determining B is based on volume changes associated with relatively large stress changes. A preliminary examination suggests that the error involved is not important in the present context.

The theoretical predictions may conveniently be investigated experimentally in two stages:

(1) Tests to investigate the influence on undrained strength of large reductions in confining pressure which do not result in pore-water tensions. These are tests in which consolidation is carried out with a substantial back pressure u_0. It follows from equation (8) that the cell pressure

may then be reduced from σ_0 to a value equal to $\sigma_0 - u_0/B$ without inducing a negative pore pressure. Within this range of total stresses the question of any departure from full saturation does not arise.

(2) Tests to investigate the influence on undrained strength of reductions in confining pressure leading to pore water tensions of various magnitudes. In the limit these will result in departures from full saturation and thus from the assumption on which the predicted values of r are based.

3. Apparatus and testing techniques

The samples for stage 1 of the investigation were consolidated and sheared in a medium pressure range (0 – 6.9 MN m^{-2} (1000 lbf in^{-2})) triaxial cell, shown diagrammatically in figure 4. The cell consists of a stainless steel cylinder 100 mm in external diameter and having a wall thickness of 10 mm. The axial load is transmitted to the sample by a honed stainless-steel ram running in a bronze bush which is rotated at 2 rev/min by a worm drive in order to eliminate the vertical component of friction on the ram. Mechanical details are given by Bishop *et al.* (1965).

The cell pressure and back pressure were provided either by a self-compensating mercury system (Bishop & Henkel 1962) or by a hydraulic constant pressure source employing a spring loaded Amsler bleed valve. The height of the building in which the laboratory is situated permitted a pressure of 6.9 MN m^{-2} (1000 lbf in^{-2}) to be obtained from the self-compensating mercury system by the addition of only three more stages, using stainless steel cylinders connected by nylon tubing jacketed in polythene as an additional precaution against rupture.

Although the mercury system is free from hunting and from significant long-term fluctuations in pressure, the multistage arrangement is difficult to operate. In stage 2 of the investigation, where a larger pressure range was required, the system incorporating the Amsler valve was generally used. These larger pressures were found to be necessary to reach the range, in the case of London Clay, in which the unconfined tests showed a significant difference in strength from the confined tests. A high pressure triaxial cell capable of operating at 69 MN m^{-2} (10 000 lbf in^{-2}) was used. This incorporates a hydraulically balanced loading ram and an internal load transducer sensitive only to the vertical component of load and unaffected by cell pressure. Some details of the apparatus are given in appendix 1.

The test specimens were 20 mm in diameter and 40 mm in height and were permitted to consolidate by drainage through the base. In the standard triaxial apparatus the sample stands on a porous ceramic disk, the lower side of which communicates through passages drilled in the pedestal to the valves controlling drainage. Undrained tests on saturated samples can be carried out satisfactorily with this system provided the cell pressure is maintained at a high enough value to ensure that the pore pressure in the sample remains positive (or in the limit does not drop below about – 83 kN m^{-2} (– 12 lbf in^{-2}) gauge pressure). At lower pressures, and in all tests at the stage when the cell pressure is dropped to zero to remove the sample for water content determination, the water in the drainage passages will cavitate and the sample will start to imbibe water from the porous ceramic disk.

This effect leads to an error in the final water content determination in all consolidated undrained tests, the magnitude of which depends on the dimensions of the sample, the coefficient of swelling of the clay and the time taken to strip down the triaxial cell. This effect also leads to an error, which is of more importance in the context of the present investigation, in the undrained strength of all samples in which significant pore-water tensions are created by reducing the

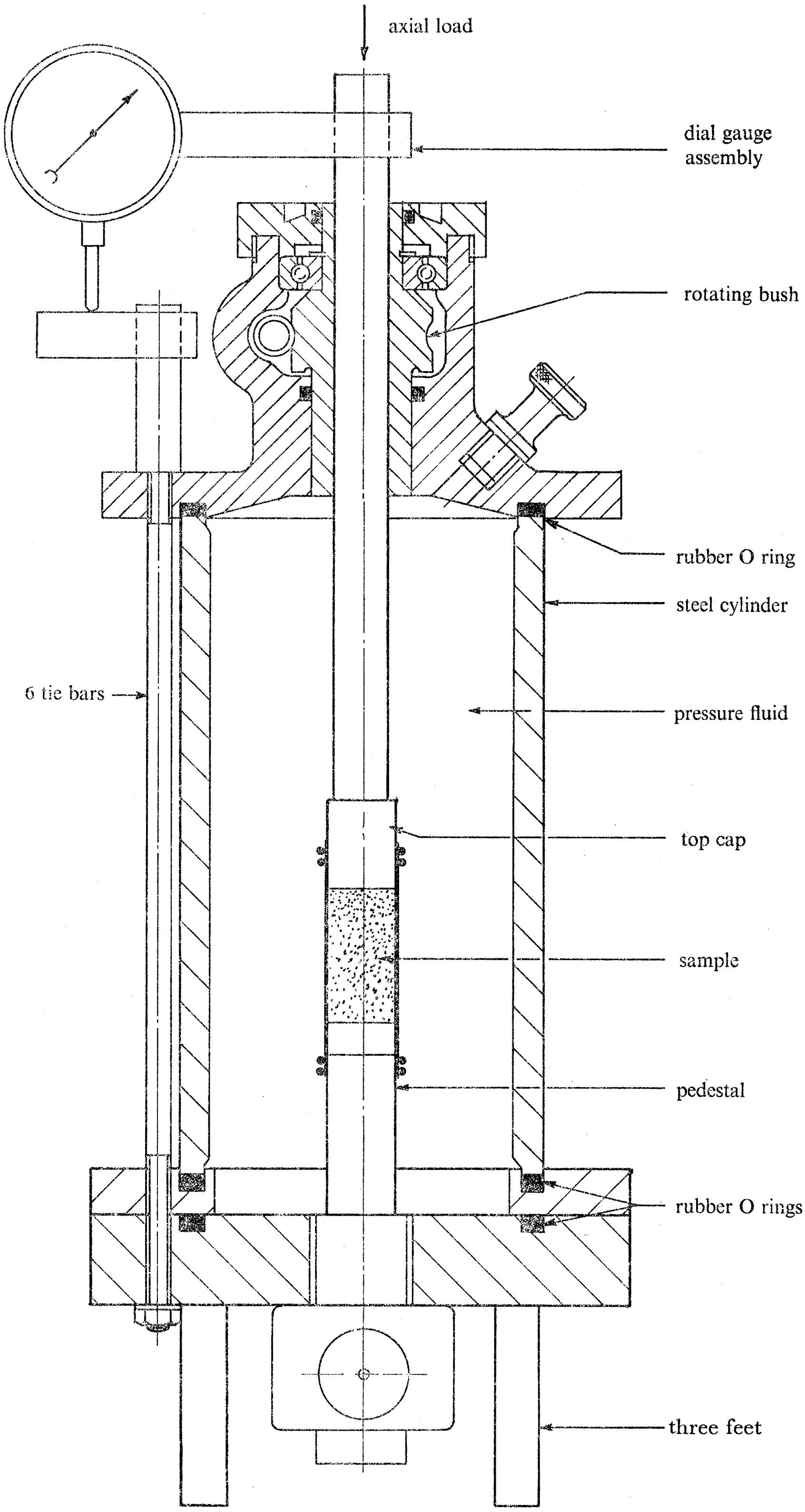

FIGURE 4. General assembly of high pressure cell.

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confining pressure. Even the use of a very short duration test (about 5 min to failure from the time at which the cell pressure was reduced), together with the use of high air entry ceramic disks and a reduction in the volume of the passage between the disk and the valves, could not bring the magnitude of this effect within acceptable limits.

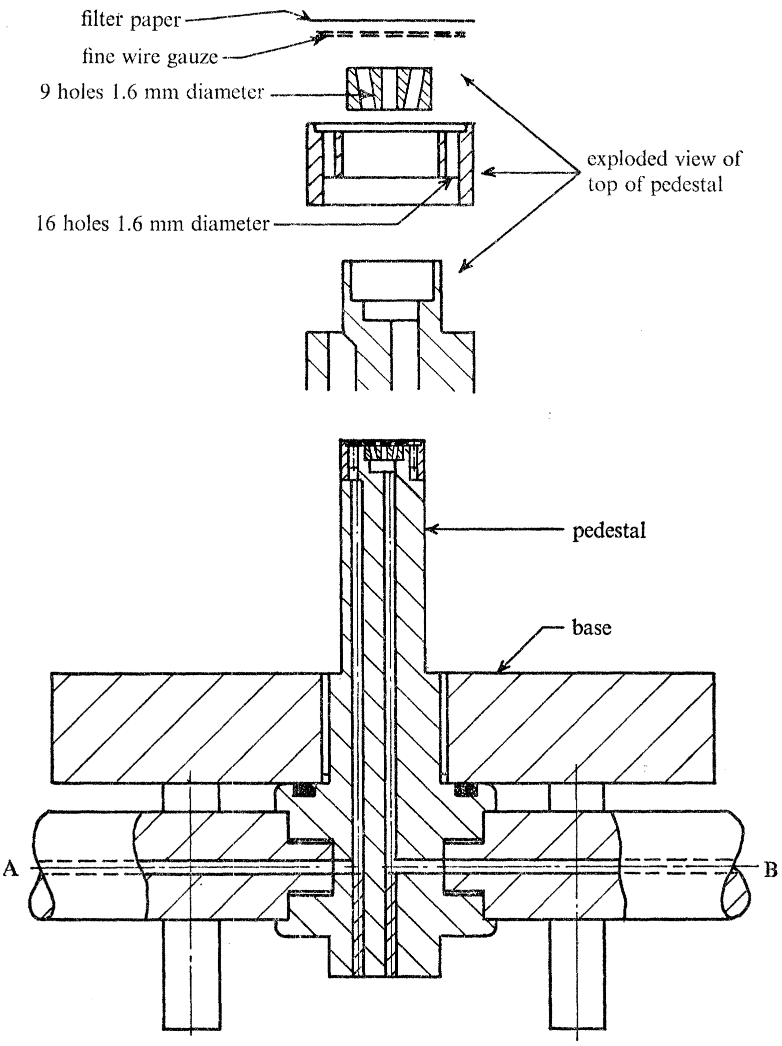

FIGURE 5. Modified triaxial cell base.

The triaxial cell base was therefore modified for stage 2 of the investigation so that, when required, all free water could be removed from below the base of the specimen by blowing dry nitrogen through the drainage system†. Details of the base are shown in figure 5. The porous ceramic disk is replaced by a disk of filter paper resting on a disk of fine wire gauze, which is recessed into the top of the pedestal so that the strands of the gauze do not puncture the rubber membrane enclosing the sample.

† A similar method was adopted by Westman (1932) in studying the effect of mechanical pressure on the moisture content of some ceramic clays. He used two porous pistons within a rigid cylinder, and removed the water from the pistons with compressed air.

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Drainage is through two groups of small diameter holes (1.6 mm diameter in the intermediate pressure range and 0.8 mm diameter in the high pressure range). One group of holes is in the stainless steel collar forming the outer annulus of the pedestal and connects with the drainage passage A on one side of the cell base. The other group of holes is in the stainless steel disk forming the centre of the pedestal and connects with the drainage passage B leading to the other side of the cell base.

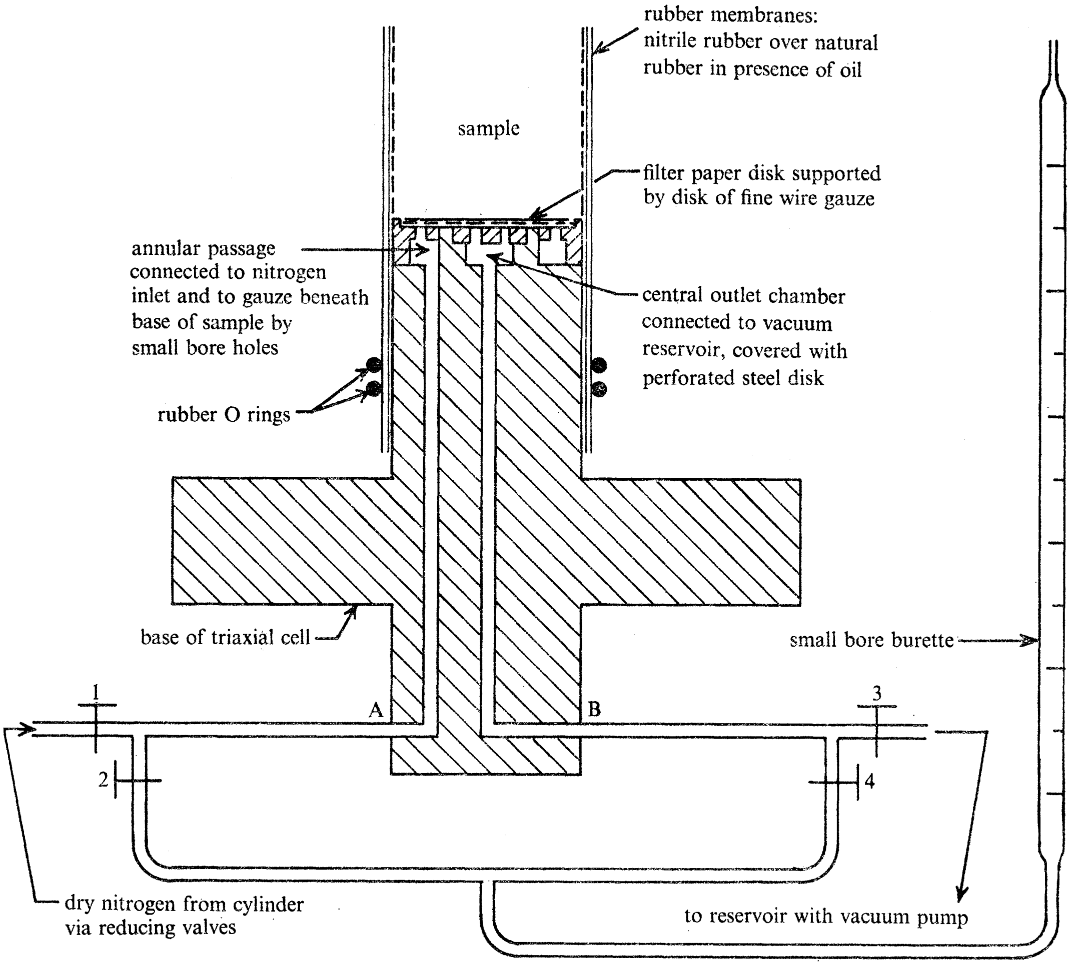

FIGURE 6. Layout of connexions to triaxial cell base for tests with high pore-water tensions (diagrammatic).

The layout of the apparatus is illustrated in figure 6. After the completion of the consolidation stage, which is carried out with a saturated pedestal and zero back pressure, the water is blown out of the connexion and burette used to measure volume change during consolidation by admitting dry nitrogen through valve 1 with valves 2 and 4 open. With valves 2 and 4 closed dry nitrogen is circulated through valve 1 to the outer annulus of the base of the sample and exhausted through the central disk to valve 3 which is opened to a reservoir connected to a vacuum pump. By admitting the nitrogen at a pressure of a little under one atmosphere (gauge) at valve 1 the base of the sample is maintained at atmospheric pressure.

The time required to remove all free water from the connecting passages and from the gauze and filter paper was determined by trial and error, and proved to be about 6 min. It was considered preferable to risk slight drying of the base of the specimen, which would not increase the measured compression strength due to the ratio of height to diameter of 2, rather than to leave free water at the base which would initiate base failure at a reduced strength.†

In order to obtain the correct moisture content values for the samples tested in the positive pore pressure range (for which undrained testing necessitated a saturated pedestal with valves 1, 2, 3 and 4 closed), an estimate of the final pore pressure was made by using equation (10) and an assumed value of A_f. The cell pressure was then dropped by an equal amount after failure, to bring the final pore pressure to zero (for $B = 1$). The system was then de-watered with dry nitrogen as above before dropping the cell pressure to zero for the removal of the sample. The close agreement apparent in table 3 between the water contents observed in the high pressure range on the unconfined samples (base de-watered before the compression test) and the confined samples (base de-watered after the compression test) indicates the success of this technique.

To ensure uniform initial conditions for all the test specimens in each series a carefully mixed clay slurry was consolidated in a 230 mm diameter oedometer to form a disk of clay about 50 mm in thickness of a strength and moisture content convenient for sample preparation. The samples were cut from the disk by means of a sharpened thin walled tube and trimmed to length before being placed in the triaxial cell.

The use of small diameter samples permitted a large series of tests to be performed on specimens cut from the same disk. To minimize handling, the disk was cut into a number of sectors, each of which was waxed, wrapped in metal foil and stored at a constant temperature until required.

The use of samples for smaller linear dimensions also brought the time required for consolidation within acceptable limits. This time depends on the square of the drainage path and owing to the necessity of de-watering the drainage system for the unconfined tests the usual expedients for shortening the drainage path of double end drainage and radial drainage could not be applied without undue complications.

4. Materials tested

The two materials used in this investigation were blue London Clay and Kaolin. The results of various classification tests are given in table 1A. The blue London Clay was from a depth of about 6.1 m in the borrow pit at Wraysbury Reservoir in Middlesex. The London Clay was deposited under marine conditions in the Eocene period. Tests reported by Bishop *et al.* (1965). suggest a preconsolidation load in this area of 4.1 MN m^{-2} (600 lbf in^{-2}). The natural water content of the clay at this level was around 29 %.

The Kaolin was a commercially prepared product mined in Cornwall and marketed as English China Clay, Spestone powder.

The initial water contents of the slurries from which the batches were prepared are given in table 1B, together with the consolidation pressures used. The batch of London Clay on which the first series of tests were performed was not allowed to swell during unloading. To reduce the pore water tensions and any consequent tendency towards partial saturation, the second batch of London Clay and the Kaolin were allowed to swell to relatively low effective stresses before storage. The higher consolidation pressures used were chosen to reduce the volume changes in the subsequent triaxial consolidation stage which might otherwise result in sample distortion when very high consolidation pressures were used.

† To determine the water content the sample was generally cut into 5 horizontal slices, and the value of the bottom slice recorded but not included in the average.

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Table 1a. Classification tests

material	liquid limit (%)	plastic limit (%)	plasticity index (%)	clay fraction (%)	activity†	relative density of particles
London Clay (1)	74	28	46	57	0.81	} 2.
London Clay (2)	78	28	50	55	0.91	
Kaolin	70	40	30	68	0.44	2.64

† Plasticity index/percentage clay.
Mineralogy of London Clay: illite with kaolinite, montmorillonite and chlorite.

Table 1b. Consolidation of clay before sampling

material	maximum vertical stress		final vertical stress		water content	
	MN m^{-2}	lbf in^{-2}	MN m^{-2}	lbf in^{-2}	initial (%)	final (%)
London Clay (1)	0.28	41	0.28	41	112	43
London Clay (2)	1.10	160	0.07	10	77.5	34.2
Kaolin	1.10	160	0.02	3	71.5	42.6

5. Test results

(a) *Stage* 1: *tests to investigate the influence on undrained strength of reductions in confining pressure which do not result in negative pore water pressures*

Samples for this series of tests were cut from batch 1 of the London Clay and were consolidated with a cell pressure $\sigma_3 = 6.9$ MN m^{-2} (1000 lbf in^{-2}) and an initial pore water pressure $u = 3.4$ MN m^{-2} (500 lbf in^{-2}). The samples were then tested, with the pedestal saturated and the drainage valves closed, at cell pressures of 6.9, 6.2, 5.5, 4.8, 4.1 and 3.4 MN m^{-2} (1000, 900, 800, 700, 600 and 500 lbf in^{-2}). The rate of axial strain approximated to $1\frac{1}{2}$ % per minute.

The results are given in table 2 and are plotted in figure 7.

Table 2. Results of tests on London Clay (series 1)

Test no.	consolidation pressures σ_3 MN m^{-2}	consolidation pressures σ_3 lbf in^{-2}	consolidation pressures u MN m^{-2}	consolidation pressures u lbf in^{-2}	values of σ_3 during compression test MN m^{-2}	values of σ_3 during compression test lbf in^{-2}	undrained strength c_u MN m^{-2}	undrained strength c_u lbf in^{-2}	c_u/p
1	6.9	1000	3.4	500	6.9	1000	0.673	97.6	0.195
2	6.9	1000	3.4	500	6.2	900	0.665	96.4	0.193
3	6.9	1000	3.4	500	5.5	800	0.665	96.5	0.193
4	6.9	1000	3.4	500	4.8	700	0.667	96.8	0.194
5	6.9	1000	3.4	500	4.1	600	0.657	95.3	0.191
6	6.9	1000	3.4	500	3.4	500	0.657	95.3	0.191

Although there is some scatter in the test results it will be seen that there is a small but significant decrease in strength, amounting to about 2.1 % for a decrease in total stress of 3.4 MN m^{-2} (500 lbf in^{-2}) which is equal to the effective consolidation pressure $p(=(\sigma_3)_0-(u)_0$ to a very close approximation). This decrease corresponds to a slope of 0.2° for the Mohr envelope for undrained strength plotted against total stress.

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This reduction in strength may be compared with the value predicted by equation (17) by substituting the following values:

$$m = 0.5$$

$$B = 0.983$$

$$-\Delta\sigma = \sigma_0 - u_0 = 3.4\ \mathrm{MN\,m^{-2}}\ (500\ \mathrm{lbf\,in^{-2}})$$

giving $$r = 0.86\,\%.$$

This value is smaller than the observed decrease in strength, but in view of the small values of both the predicted and observed values of r in relation to the scatter of the strength measurements plotted in figure 7, it is doubtful whether this difference is significant.

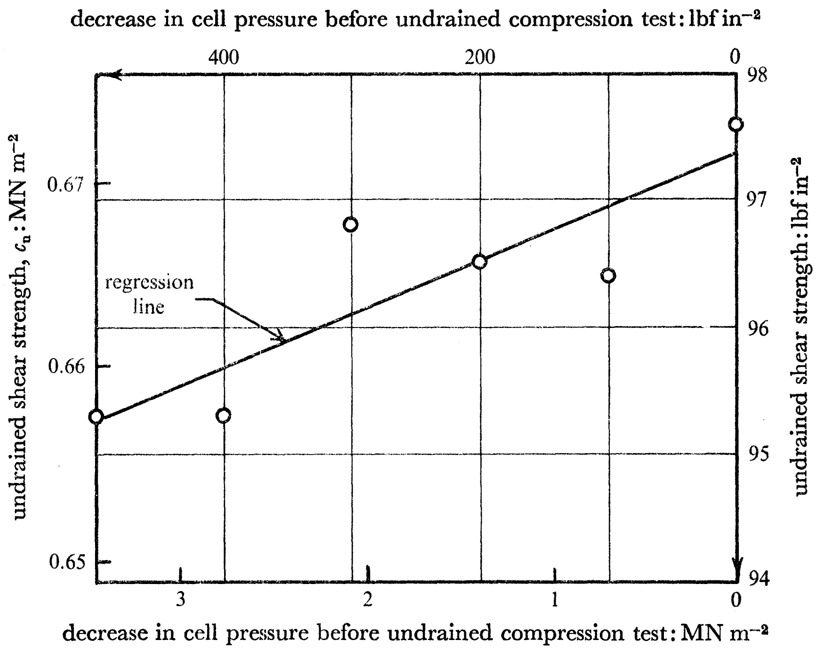

FIGURE 7. Relation between undrained shear strength and decrease in cell pressure after completion of consolidation stage: Blue London Clay, consolidated under $\sigma_3 = 6.9\ \mathrm{MN\,m^{-2}}$ (1000 lbf in^{-2}), $u = 3.4\ \mathrm{MN\,m^{-2}}$ (500 lbf in^{-2}).

(b) *Stage 2: tests to investigate the influence on undrained strength of reductions in confining pressure leading to pore water tensions*

Samples for this series of tests were cut from batch 2 of the London Clay and from the Kaolin. The samples were consolidated with a range of cell pressures 6.9–62.1 MN m^{-2} (1000–9000 lbf in^{-2}) for the London Clay and 0.86–6.9 MN m^{-2} (125–1000 lbf in^{-2}) for the Kaolin but with zero back pressure in all cases. The samples were then tested either with the cell pressure equal to the consolidation pressure ($\sigma_3 = p$) or unconfined ($\sigma_3 = 0$), the pedestal being de-watered at the appropriate stage as described in § 3. The rate of axial strain was approximately 2 % per minute.

The results are given in tables 3 and 5, but will be described separately in the next two sections.

(i) *Tests on London Clay*

Two preliminary tests on remoulded London Clay (table 3, 1B and 2B) had indicated that the ratio c_u/p for unconfined specimens did not differ significantly from that for confined specimens for a consolidation pressure of 6.9 MN m^{-2} (1000 lbf in^{-2}) and that much higher consolidation pressures must therefore be used for this stage of the investigation. Subsequent tests were carried out on batch 2 of the London Clay with a lower initial water content, in order to reduce the magnitude of the volume change and associated distortion, on subsequent consolidation in the triaxial cell.

The two most notable effects of high pore-water tensions on the strength and deformation

Table 3. Results of tests on London clay (series 2)

test no.	consolidation pressures σ_3 MN m^{-2}	lbf in^{-2}	u	water contents initial w (%)	final w (%)	tests with $\sigma_3 = p$: c_u MN m^{-2}	lbf in^{-2}	c_u/p	tests with $\sigma_3 = 0$: c_u MN m^{-2}	lbf in^{-2}	c_u/p
1B†	6.9	1000	0	54.6	19.5	—	—	—	1.44	209	0.209
2B†	6.9	1000	0	53	19.3	—	—	—	1.45	210	0.210
3B	6.9	1000	0	36.2	19.7	—	—	—	1.37	198	0.198
4B	6.9	1000	0	32.4	19.9	1.39	201	0.201	—	—	—
5B	—	—	—	—	—	—	—	—	—	—	—
6B	6.9	1000	0	35.4	19.5	—	—	—	1.42	206	0.206
7B	6.9	1000	0	34.8	19.8	1.52	221	0.221	—	—	—
8B	6.9	1000	0	35.5	19.6	1.38	200	0.200	—	—	—
9B	—	—	—	—	—	—	—	—	—	—	—
10B	6.9	1000	0	33.4	20.0	1.40	203	0.203	—	—	—
11B	6.9	1000	0	33.5	19.5	1.43	207	0.207	—	—	—
12B‡	6.9	1000	0	33.6	—	—	—	—	—	—	—
	3.4	500	0	—	21.8¶	—	—	—	1.04	151	0.302
13B	20.7	3000	0	33.6	13.8	4.40	638	0.213	—	—	—
14B	20.7	3000	0	34.5	13.8	—	—	—	4.00	580	0.193
15B§	6.9	1000	0	33.1	—	—	—	—	—	—	—
	0.69	100	0	—	—	—	—	—	—	—	—
	6.9	1000	0	—	17.8‡‡	1.66	241	0.241	—	—	—
16B	20.7	3000	0	36.7	13.9	—	—	—	4.09	593	0.198
17B	20.7	3000	0	34.7	13.4	4.07	590	0.197	—	—	—
18B	20.7	3000	0	33.5	13.4	4.41	640	0.213	—	—	—
19B	62.1	9000	0	33.5	10.2	11.9	1723	0.191	—	—	—
20B‖	6.9	1000	0	33.9	19.7¶	—	—	—	1.35	196	0.196
21B	62.1	9000	0	33.9	10.1	—	—	—	6.76	980††	0.109
22B	62.1	9000	0	33	10.1	—	—	—	7.03	1020††	0.113
23B	—	—	—	—	—	—	—	—	—	—	—
24B	41.4	6900	0	33.2	11.0	—	—	—	5.69	825††	0.137
25B	41.4	6000	—	33.5	10.8	9.14	1325	0.221	—	—	—
26B	6.9	1000	0	35.2	—	—	—	—	—	—	—
	0.69	100	0	—	—	—	—	—	—	—	—
	6.9	1000	0	—	19.1	—	—	—	—	—	—

† Preliminary series with higher initial water content.
‡ Sample consolidated to 6.9 MN m^{-2} and allowed to swell under 3.4 MN m^{-2} before testing.
§ Sample allowed to swell and then reconsolidated before testing.
‖ Sample allowed to stand unconfined for 35 min before testing.
¶ Calculated from volume change.
†† The expansion on stress release and reduction in volume under the axial stress were not measured during these tests and are significant only for the samples which departed from full saturation. At the maximum consolidation pressure the tabulated value may represent an overestimate of the strength by about $2\frac{1}{2}$ %.
‡‡ The final water content determination appears to be in error and should approximate to that of 26B.

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characteristics of clay are illustrated very clearly by the series of stress–strain curves for progressively increasing consolidation pressure (figures 8–11). The stress–strain curves for samples consolidated with a cell pressure $\sigma_0 = 6.9\,\mathrm{MN\,m^{-2}}$ ($1000\,\mathrm{lbf\,in^{-2}}$) and a pore water pressure $u_0 = 0$ are illustrated in figure 8. It will be seen that the samples tested with the cell pressure maintained at $6.9\,\mathrm{MN\,m^{-2}}$ (and thus with the pore pressure zero at the beginning of the test and positive at failure) show a rounded stress–strain curve with a relatively gradual reduction in stress after failure. In contrast the samples tested after reducing the cell pressure to zero (and thus inducing a negative pore pressure of approximately $6.9\,\mathrm{MN\,m^{-2}}$ before the beginning of the undrained compression test) retained approximately the same peak value of strength but showed a rapid reduction in strength after failure amounting almost to brittle failure.

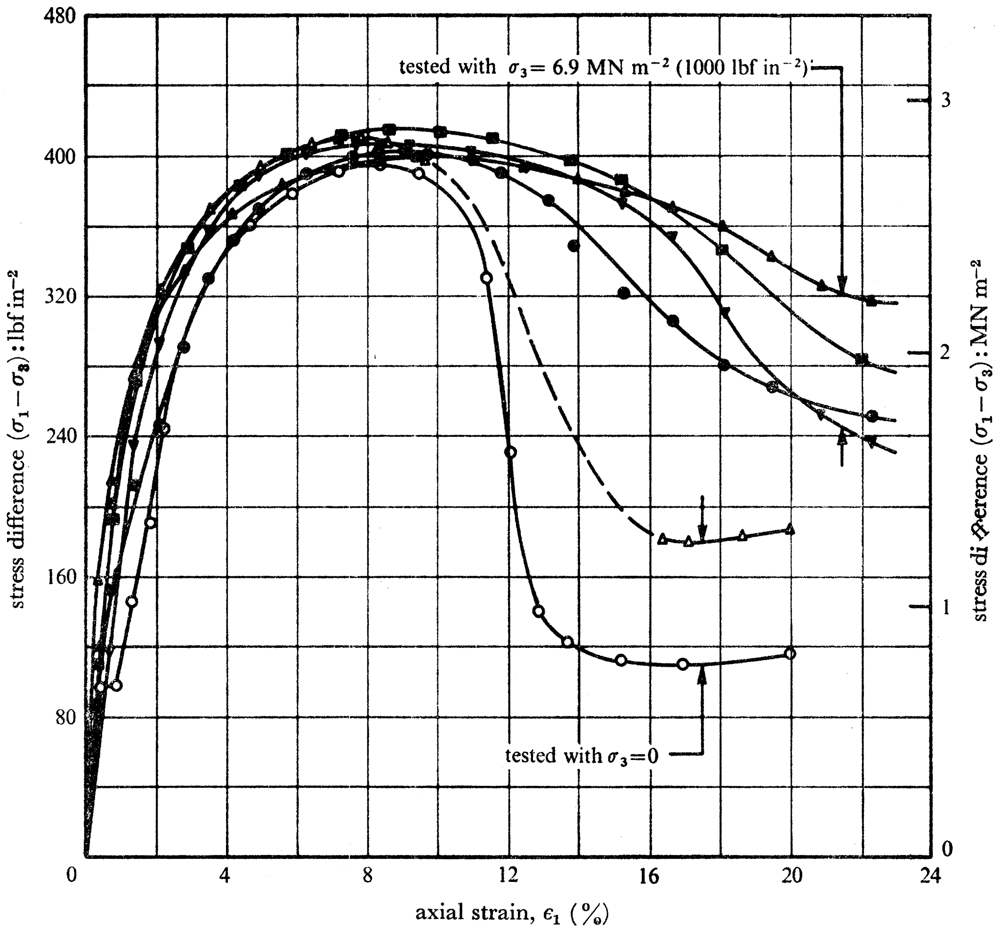

FIGURE 8. Comparison of confined and unconfined compression tests on samples of London Clay consolidated with $\sigma_3 = 6.9\,\mathrm{MN\,m^{-2}}$ ($1000\,\mathrm{lbf\,in^{-2}}$) and $u = 0$.

The fractured specimens fell apart on removal from the enclosing membrane in the case of all specimens tested under high negative pore pressures, whereas those tested under positive pore pressures could not be separated by hand along the rupture surface which was visible on the surface of the specimen. The failure mechanisms are illustrated in figure 12, plate 17, by photographs of samples subjected to the highest consolidation pressure. The corresponding curves for a consolidation pressure of $20.7\,\mathrm{MN\,m^{-2}}$ ($3000\,\mathrm{lbf\,in^{-2}}$) show the same general trends (figure 9), though there is a small but significant reduction in peak strength accompanied by a marked increase in the brittleness of the specimens tested under the condition of high negative pore pressure.

The stress–strain curves for a consolidation pressure of 41.4 MN m^{-2} (6000 lbf in^{-2}) show a very substantial reduction in peak strength in the specimen tested with a high induced negative pore pressure accompanied again by a very brittle type of failure (figure 10).

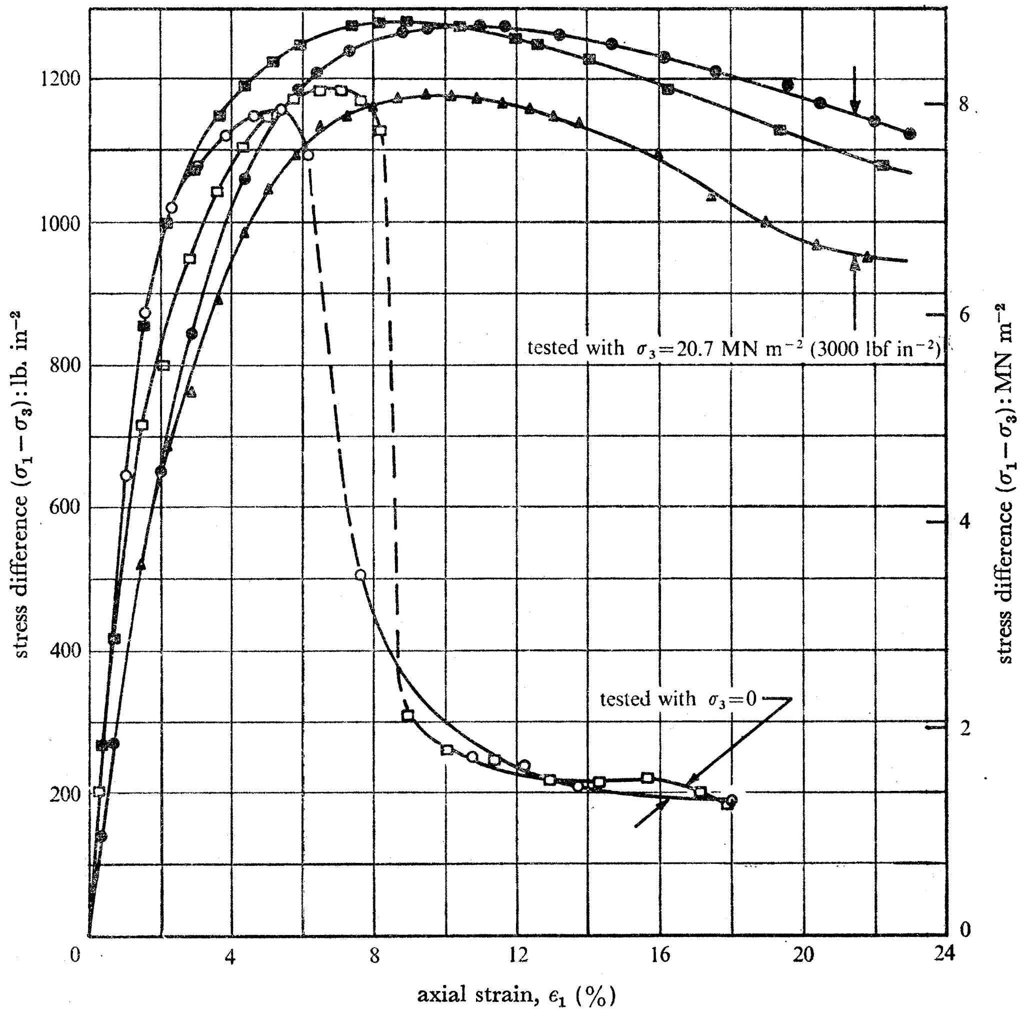

FIGURE 9. Comparison of confined and unconfined compression tests on samples of London Clay consolidated with σ_3 = 20.7 MN m^{-2} (3000 lbf in^{-2}) and u = 0.

The corresponding curves (figure 11) for a consolidation pressure of 62.1 MN m^{-2} (9000 lbf in^{-2}) (the maximum pressure at which the load transducer could be used) show an even larger percentage reduction in compression strength (42 %) and a very brittle failure, the fractured specimen being illustrated in figure 12.

The results are summarized on graphs relating undrained shear strength c_u and consolidation pressure p (figure 13), undrained shear strength c_u and water content as tested w (figure 14) and percentage loss in strength r against reduction in confining pressure (figure 15). It is clear from all three sets of curves that a major change in behaviour is beginning at a reduction in cell pressure of about 20.7 MN m^{-2} (3000 lbf in^{-2}). A theoretical relation between the reduction in strength r, the reduction in confining pressure $-\Delta\sigma$ and the consolidation pressure $\sigma_0 - u_0$ was obtained in §2, equation (17). In the present series of tests $u_0 = 0$, and for the samples tested unconfined the reduction in confining pressure $-\Delta\sigma = \sigma_0$. Equation (17) thus reduces to the expression:

$$r = m(1-B). \tag{17a}$$

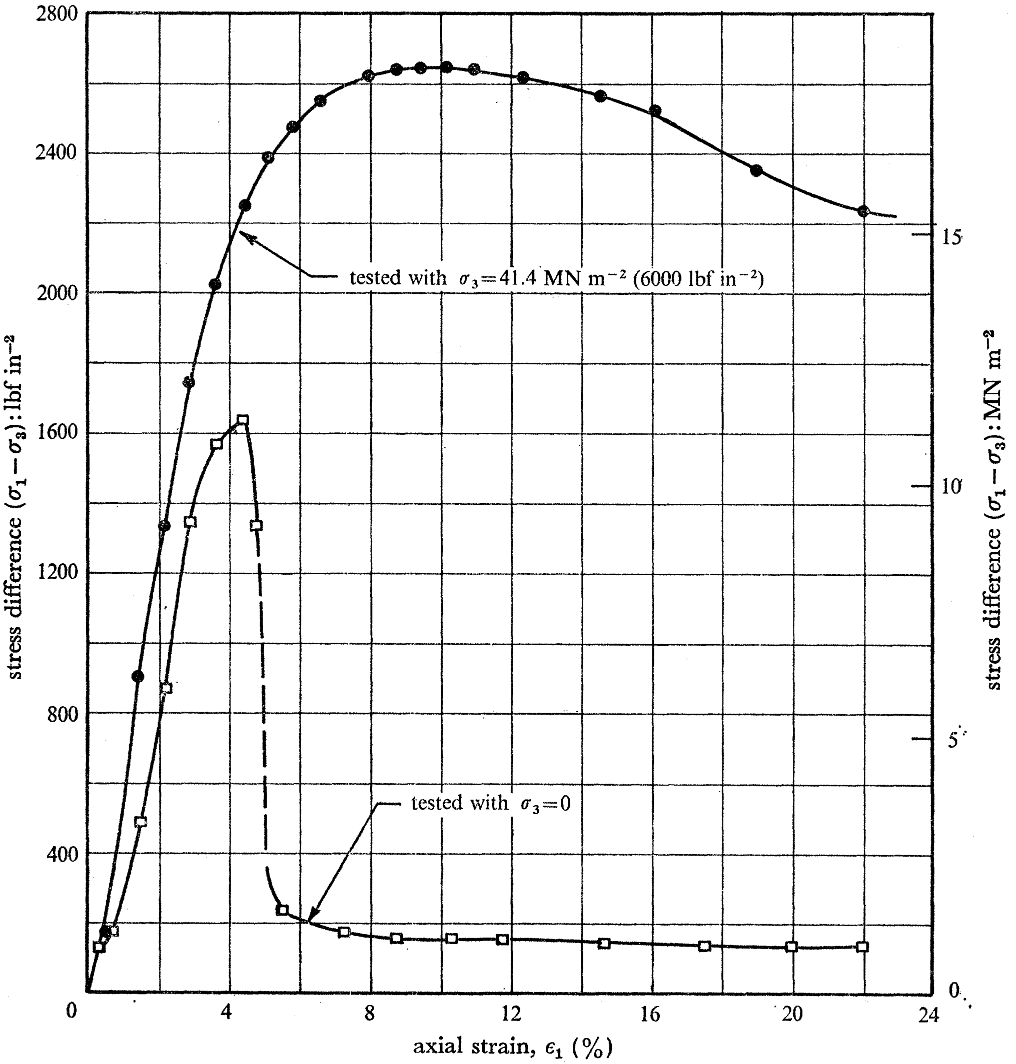

FIGURE 10. Comparison of confined and unconfined compression tests on samples of London Clay consolidated with σ_3 = 41.4 MN m^{-2} (6000 lbf in^{-2}) and u = 0.

For the very wide range of pressures used in the present series of tests, the relation between the void ratio e and the logarithm of consolidation pressure p departs substantially from a straight line (figure 16) and the compressibility falls to a value less than that of water. No simple relation can therefore be established between r and p, and the values of r must be obtained numerically for each stress level, using the more accurate expression for B (equation 2) for the higher consolidation pressures. In the calculation of B on unloading the expansibility of the soil structure is taken to bear a ratio λ to the compressibility C. In the lower stress range the value of λ is about

0.45 for London Clay consolidated from a slurry and allowed to swell under an equal all-round pressure. At high consolidation pressures this value might be expected to rise.

The values of r for the four values of consolidation pressure and for two assumptions about λ are given in table 4 and are plotted in figure 15. It will be seen that the theoretical relations between r and the reduction in confining pressure for samples remaining fully saturated are smooth curves which do not depart to any marked extent from straight lines except at the highest

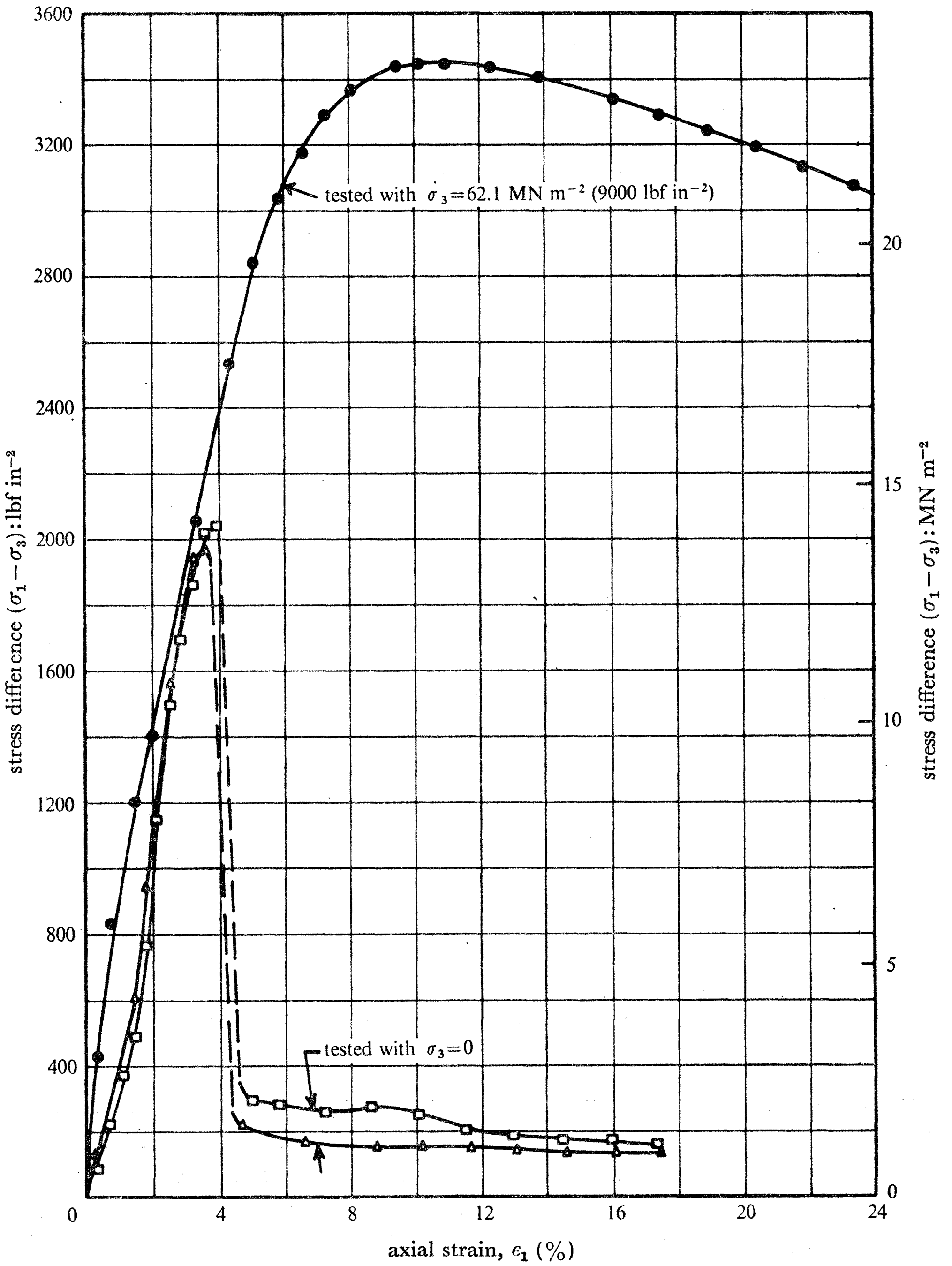

FIGURE 11. Comparison of confined and unconfined compression tests on samples of London Clay consolidated with σ_3 = 62.1 MN m^{-2} (9000 lbf in^{-2}) and u = 0.

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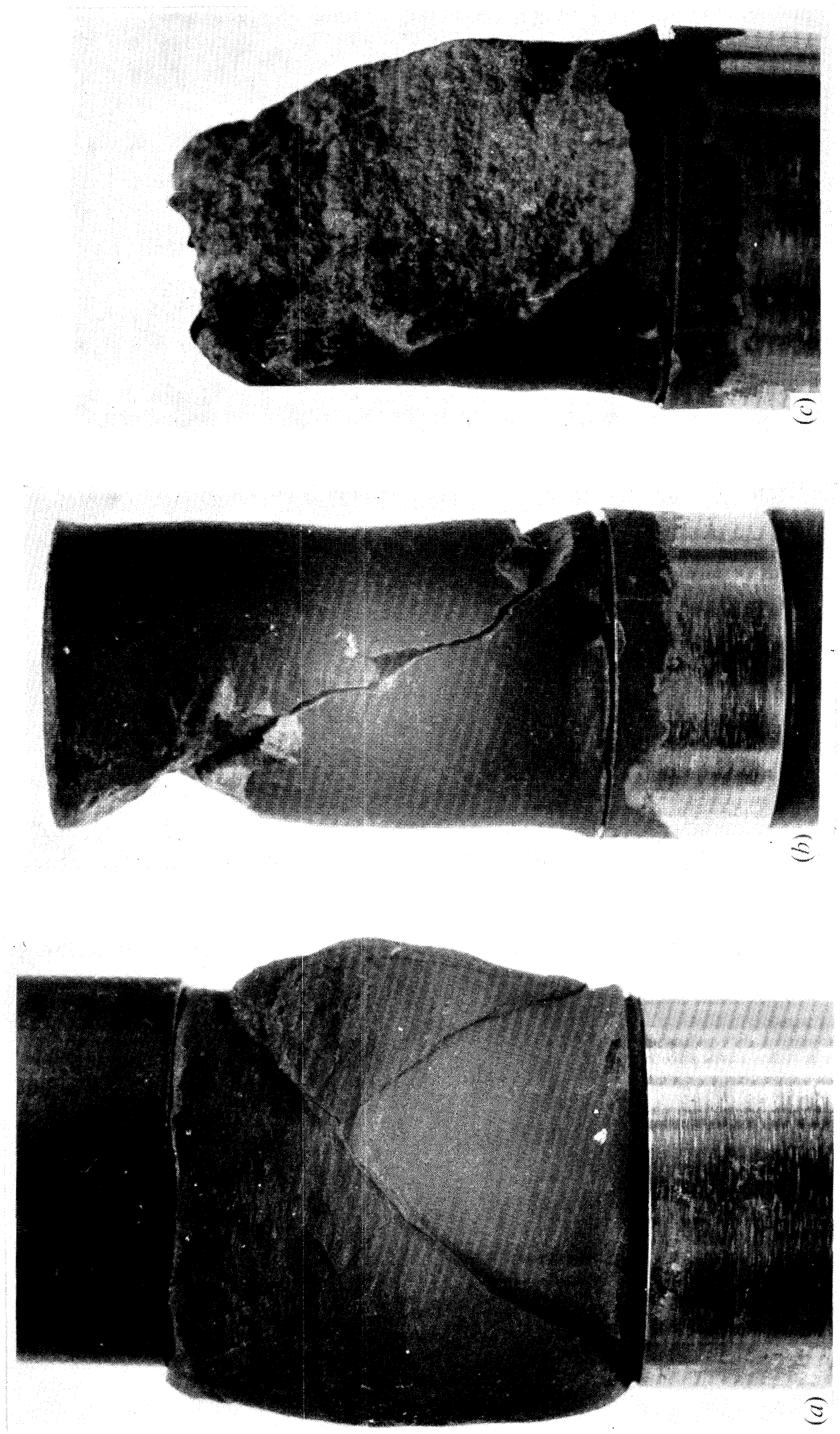

FIGURE 12. Samples consolidated at 62.1 MN m^{-2} (9000 lbf in^{-2}) and tested (*a*) with cell pressure equal to consolidation pressure (test no. 19 B) and (*b*) unconfined (test no. 21 B). Surface texture shown in (*c*) for unconfined test.

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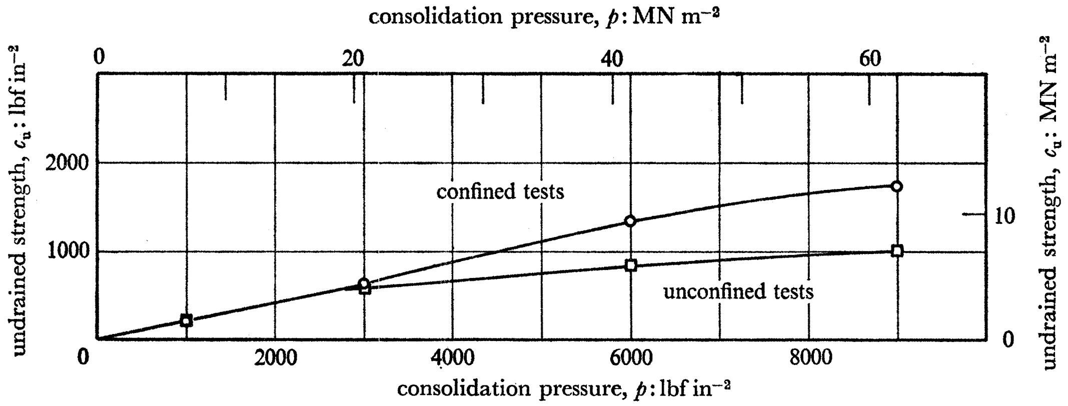

FIGURE 13. Relations between undrained strength c_u and consolidation pressure p for confined and unconfined tests on London Clay. Where a number of tests were performed at a particular pressure, the average value is plotted.

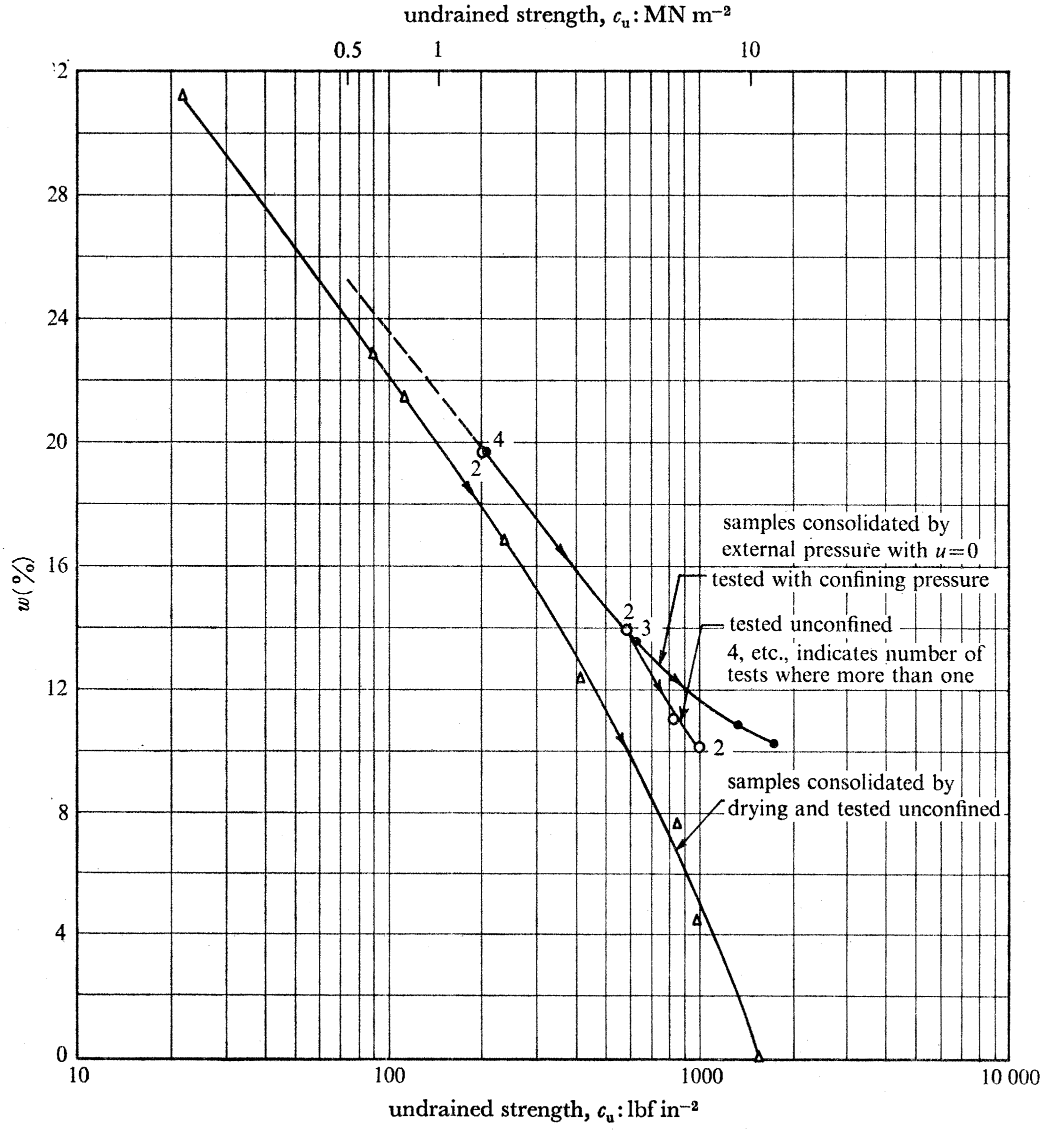

FIGURE 14. Relations between undrained strength c_u and water content w for both confined tests and unconfined tests on samples consolidated by external pressure and for unconfined tests on samples consolidated by drying: London Clay.

cell pressure. The departure of the observed line relating loss in strength to reduction in total stress from the shape of the theoretical lines† at a pressure of a little below 20.7 MN m^{-2} (3000 lbf in^{-2}) suggests that a breakdown in the condition of full saturation occurs at this point. It will be seen from table 4 that on the assumption of full saturation the predicted value of r for a reduction in total stress of 6.9 MN m^{-2} (1000 lbf in^{-2}) lies in the range 1.22–1.40 %. The observed value lies in the range 0.5–2.0 %, the latter value being obtained if one exceptionally high value is included in the average of the confined test results (see tables 3 and 7). The agreement is thus clearly within experimental error.

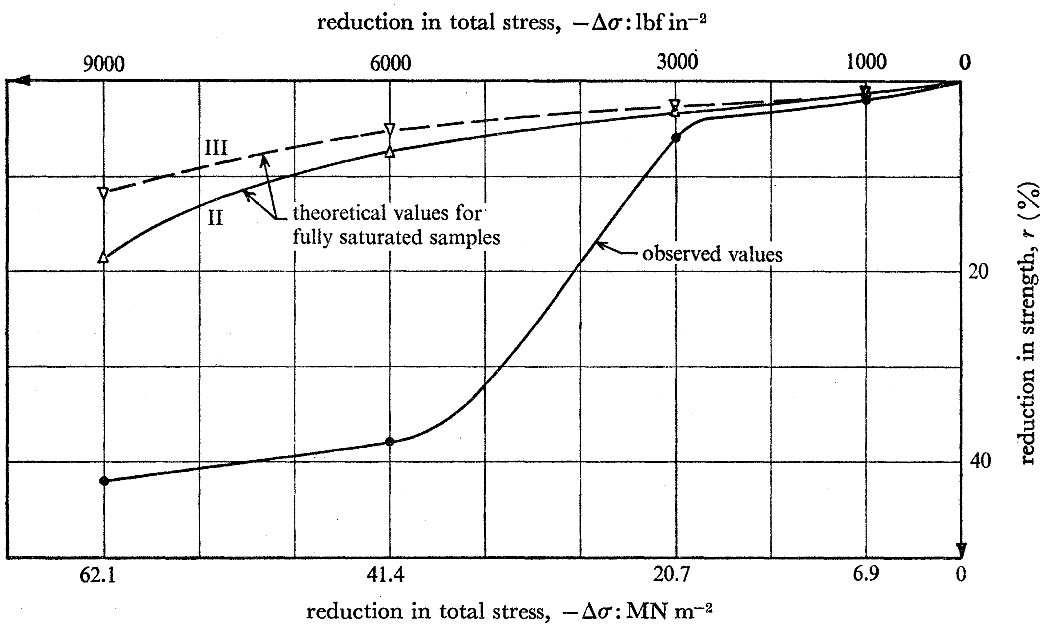

FIGURE 15. Relations between reduction in strength r and reduction in total stress $-\Delta\sigma$. A comparison between the observed values and the theoretical values based on the assumption of full saturation. Normally consolidated London Clay; initial pore-water pressure is 0.

It is also apparent from the stress–strain curves (figure 9) that a major change in the mechanism of failure is beginning to take place at this stress level. As the induced pore water tension in the unconfined samples at this consolidation pressure, if fully saturated, is $B \times p$ and thus equals approximately $0.94 \times 3000 = 19.4$ MN m^{-2} (2820 lbf in^{-2}), the breakdown might be due either to tensile failure in the pore water or due to failure of capillary menisci to prevent the invasion of the pore space by air or nitrogen in contact with the specimen or released from solution in the pore water. A series of tests was therefore carried out on Kaolin which has a radically different pore size, as is indicated by a permeability some two orders of magnitude greater than that of London Clay although the percentage of clay size particles is greater in the Kaolin (table 1A).

† If it had been assumed that a departure from unity of the value of k in equation (6) was partly responsible for the loss in strength, the shape of curves II and III would not be significantly changed. From equation (5) we would have $1-k_s = a(\tan\psi/\tan\phi')$ where a is the contact area between the particles per unit area. Since, as indicated earlier, this area is likely to be almost directly proportional to consolidation pressure (Skempton 1960), the component of r involving $1-K_s$ would likewise be almost linearly related to consolidation pressure.

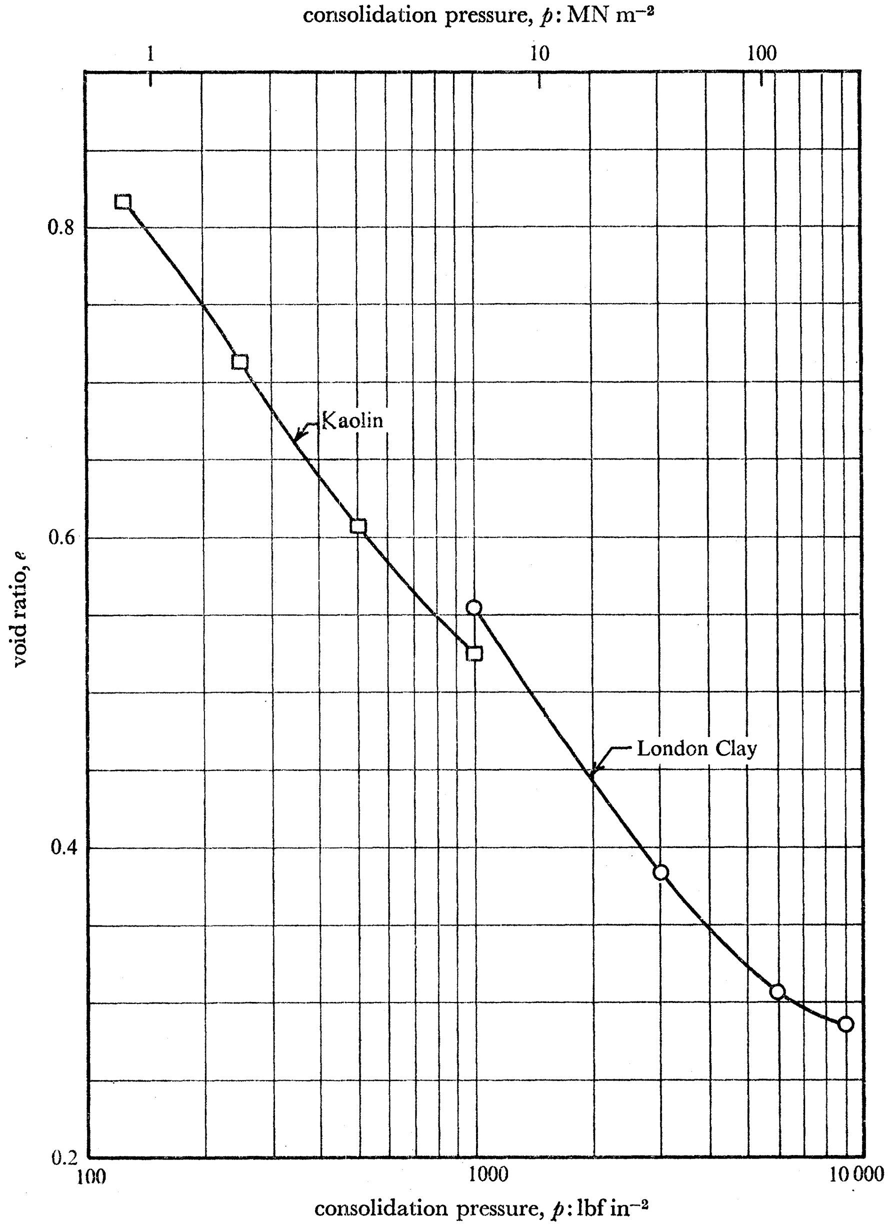

FIGURE 16. Relations between the void ratio e calculated from the final water content and the consolidation pressure p for London Clay and Kaolin.

TABLE 4. THE BASIS FOR THE DETERMINATION OF THE VALUES OF THE PARAMETER r

consolidation pressure		compressibility, C from p–e curves	expansibility, C_e from p–e curves		expansibility C_e in m^2 MN^{-1} units on basis of 3 assumptions†			porosity	values of r (%) for assumptions II and III	
N m^{-2}	lbf in^{-2}	m^2 MN^{-1}	m^2 MN^{-1}	$\lambda = C_e/C$	I	II	III	n	II	III
6.9	1000	15.8×10^{-3}	6.2×10^{-3}‡	0.39	6.2×10^{-3}	7.1×10^{-3}	6.2×10^{-3}	0.357	1.22	1.40
20.7	3000	45.0×10^{-4}	24.0×10^{-4}§	0.53	23.7×10^{-4}	20.2×10^{-4}	24.0×10^{-4}	0.277	3.10	2.80
41.4	6000	14.8×10^{-4}	—	—	12.4×10^{-4}	6.6×10^{-4}	10.0×10^{-4}	0.235	7.20	5.05
62.1	9000	45.0×10^{-5}	—	—	84.3×10^{-5}	20.2×10^{-5}	36.0×10^{-5}	0.222	18.5	11.7

Assumption: I, unloading p against e curves are straight parallel lines on an e against lg p plot; II, the value of λ is constant and al to 0.45; III, extrapolating observed C_e values using a value of λ increasing with p.

Assuming a linear relation between e and lg p on unloading from 6.9 to 0.69 MN m^{-2} (1000 – 100 lbf in^{-2}).

Extrapolated from volume change observed over range 19.7–17.6 MN m^{-2} (2854–2554 lbf in^{-2}).

(ii) *Tests on Kaolin*

The second test on Kaolin (table 5, test 2D) gave a clear indication that the breakdown pressure for Kaolin was well below 6.9 MN m^{-2} (1000 lbf in^{-2}). The series of tests were therefore carried out for consolidation pressures in the range 0.86–6.9 MN m^{-2} (125–1000 lbf in^{-2}). The samples generally showed substantially larger strains at failure than observed for London Clay, and several samples showed a tendency to tilt associated with a reduced strength at failure.

Table 5. Results of tests on Kaolin

	consolidation pressures			water contents		tests with $\sigma_3 = p$			tests with $\sigma_3 = 0$		
	σ_3			initial	final	c_u			c_u‡		
test no.	MN m^{-2}	lbf in^{-2}	u	w (%)	w (%)	MN m^{-2}	lbf in^{-2}	c_u/p	MN m^{-2}	lbf in^{-2}	c_u/p
1D	6.9	1000	0	44.2	20.0	1.68	243	0.243	—	—	—
2D	6.9	1000	0	43.0	19.9	—	—	—	0.64	93	0.093
3D	6.9	1000	0	43.2	20.1	—	—	—	0.54	78	0.078
4D	3.4	500	0	42.2	23.2	—	—	—	0.52	75	0.150
5D	6.9	1000	0	42.7	19.6	—	—	—	0.61	88	0.088
6D†	1.7	250	0	42.9	27.2	—	—	—	0.38	55.5	0.222
7D	1.7	250	0	43.0	26.9	—	—	—	0.42	61.0	0.244
8D	1.7	250	0	42.0	27.0	0.49	71.0	0.284	—	—	—
9D	1.7	250	0	41.2	27.1	0.43	62.5	0.250	—	—	—
10D	1.7	250	0	42.0	26.8	0.49	71.0	0.284	—	—	—
11D†	3.4	500	0	42.8	22.5	0.74	108	0.216	—	—	—
12D†	3.4	500	0	42.9	23.2	0.70	101	0.202	—	—	—
13D	—	—	—	—	—	—	—	—	—	—	—
14D	—	—	—	—	—	—	—	—	—	—	—
15D	3.4	500	0	42.8	23.1	0.90	130	0.260	—	—	—
16D	0.86	125	0	42.5	30.8	—	—	—	0.25	36.3	0.290
17D	0.86	125	0	42.1	30.5	0.24	35.0	0.280	—	—	—
18D	0.86	125	0	42.6	31.3	—	—	—	0.22	32.0	0.256

† Samples tilted during compression test.

‡ The expansion on stress release and reduction in volume under the axial stress were not measured during these tests and are significant only for the samples which departed from full saturation. At the maximum consolidation pressure the tabulated value may represent an over-estimate of the strength by about 7 %.

These latter tests have been assumed to be unrepresentative. The stress–strain curves are presented in figures 17–20. It should be noted that the lowest consolidation pressure $\sigma_3 = 0.86$ MN m^{-2} (125 lbf in^{-2}) lies a little below the maximum vertical stress applied to the batch during preparation in the oedometer. Samples consolidated at this pressure are therefore lightly overconsolidated, though the increase in the value of c_u/p is small (a maximum value of 0.290 for one unconfined test as against a maximum value of 0.284 at a consolidation pressure of 1.72 MN m^{-2} (250 lbf in^{-2})). The unconfined samples at the lowest consolidation pressure do not show a significant decrease in undrained strength (or in the ratio c_u/p), but the rapid drop in strength after failure noted with London Clay is not apparent at this low stress level. At a consolidation pressure of 1.72 MN m^{-2} (250 lbf in^{-2}) a reduction in strength in the unconfined specimens is already becoming apparent (figure 18). The rapid post-peak drop in strength has also appeared.

As the consolidation pressure is raised to 3.45 MN m^{-2} (500 lbf in^{-2}), the divergence in strength becomes very marked and the brittle failure of the unconfined specimens is becoming more like that observed on the London Clay specimens with a consolidation pressure an order of magnitude higher (figure 19).

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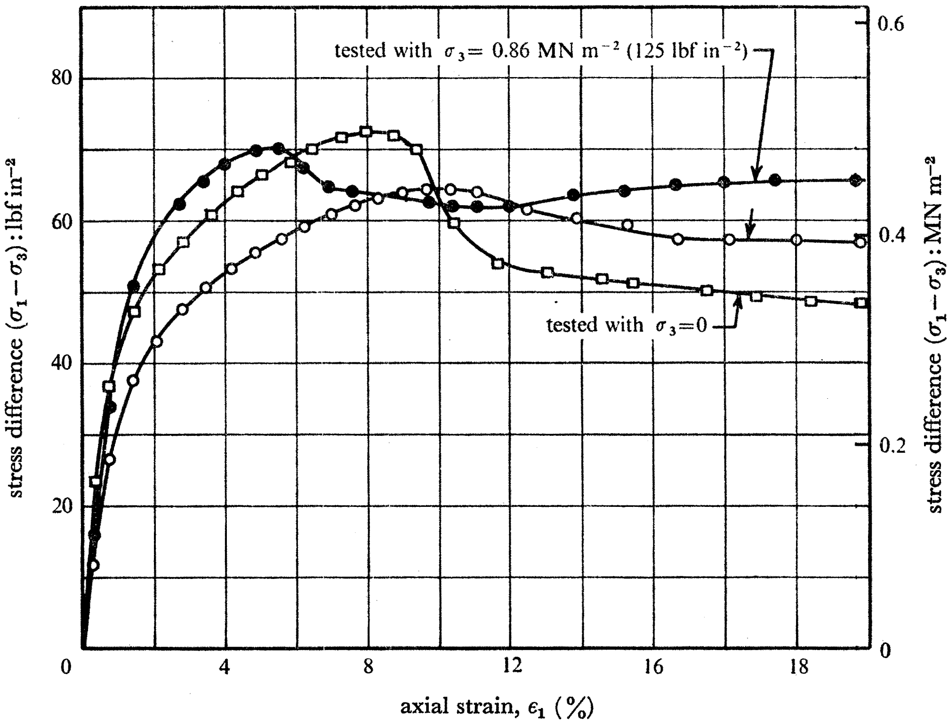

FIGURE 17. Comparison of confined and unconfined compression tests on samples of Kaolin consolidated with σ_3 = 0.86 MN m^{-2} (125 lbf in^{-2}) and u = 0.

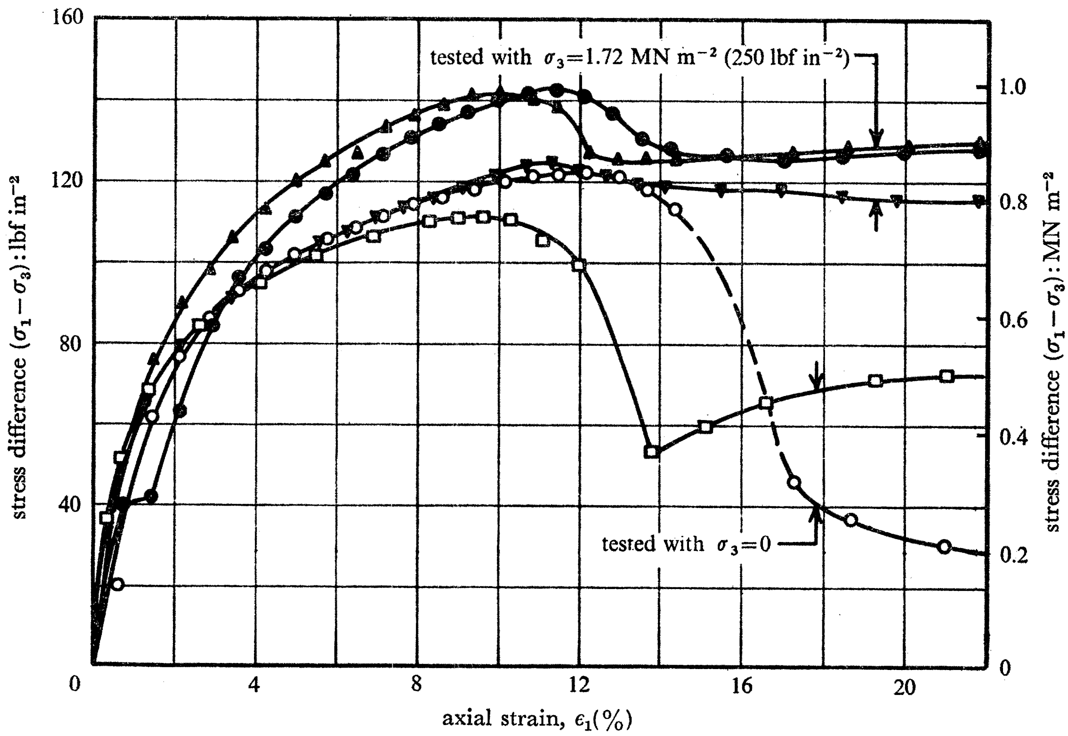

FIGURE 18. Comparison of confined and unconfined compression tests on samples of Kaolin consolidated with σ_3 = 1.72 MN m^{-2} (250 lbf in^{-2}) and u = 0.

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At a consolidation pressure of 6.9 MN m^{-2} (1000 lbf in^{-2}), the divergence in strength is even more marked (figure 20), the loss in strength on the removal of the confining pressure being 65%. The brittleness of the relatively low strength unconfined samples is, however, not quite as marked as that of the unconfined samples of London Clay, which have been subjected to higher consolidation pressure (figures 10 and 11).

The results are summarized, as for the London Clay, on graphs relating c_u and p (figure 21), c_u and w (figure 22) and percentage loss in strength r and reduction in confining pressure (figure 23). It is apparent from the three sets of curves that the divergence in strength begins at a reduction in confining pressure of about 1.38 MN m^{-2} (200 lbf in^{-2}).

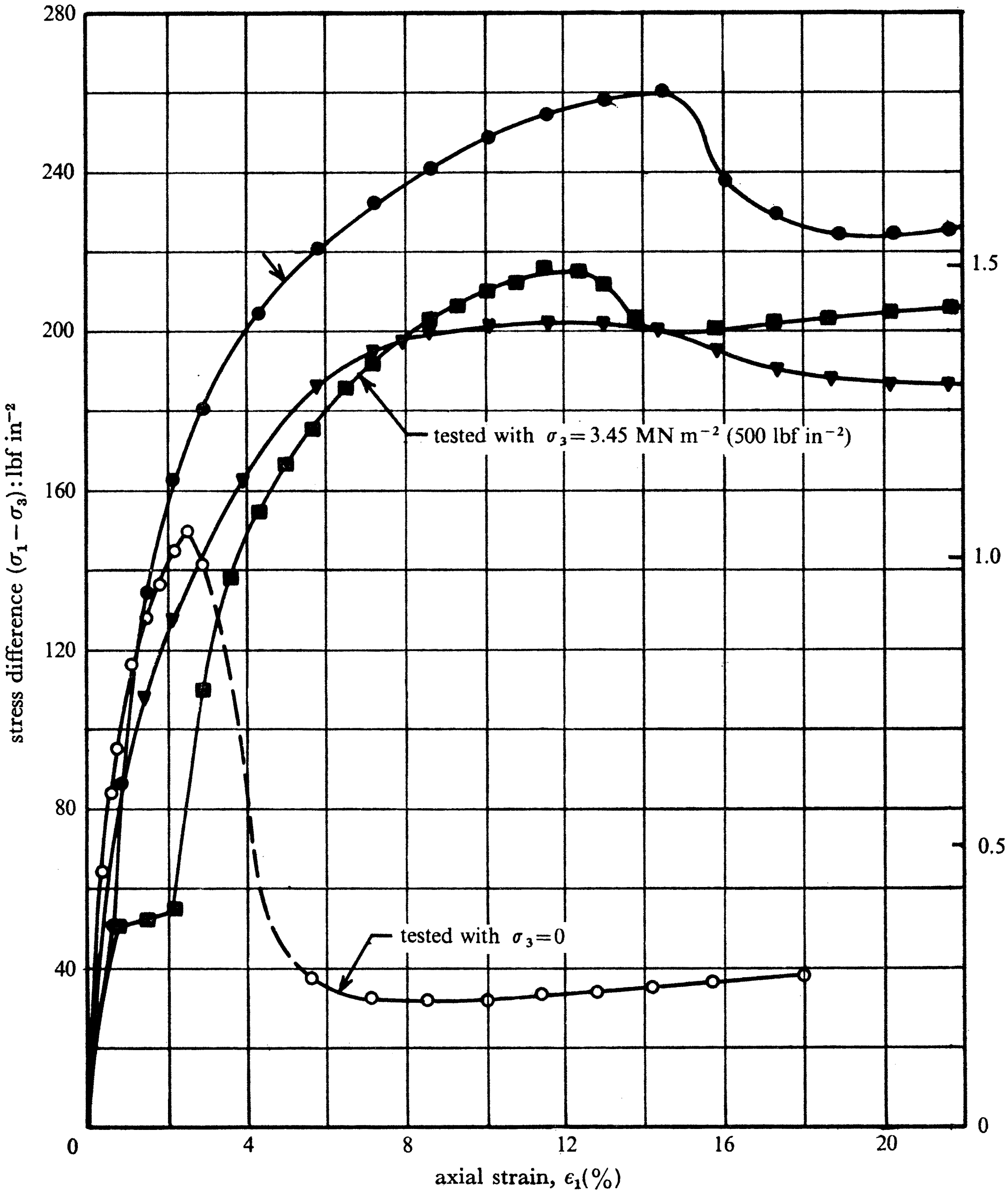

FIGURE 19. Comparison of confined and unconfined compression tests on samples of Kaolin consolidated with σ_3 = 3.45 MN m^{-2} (500 lbf in^{-2}) and u = 0.

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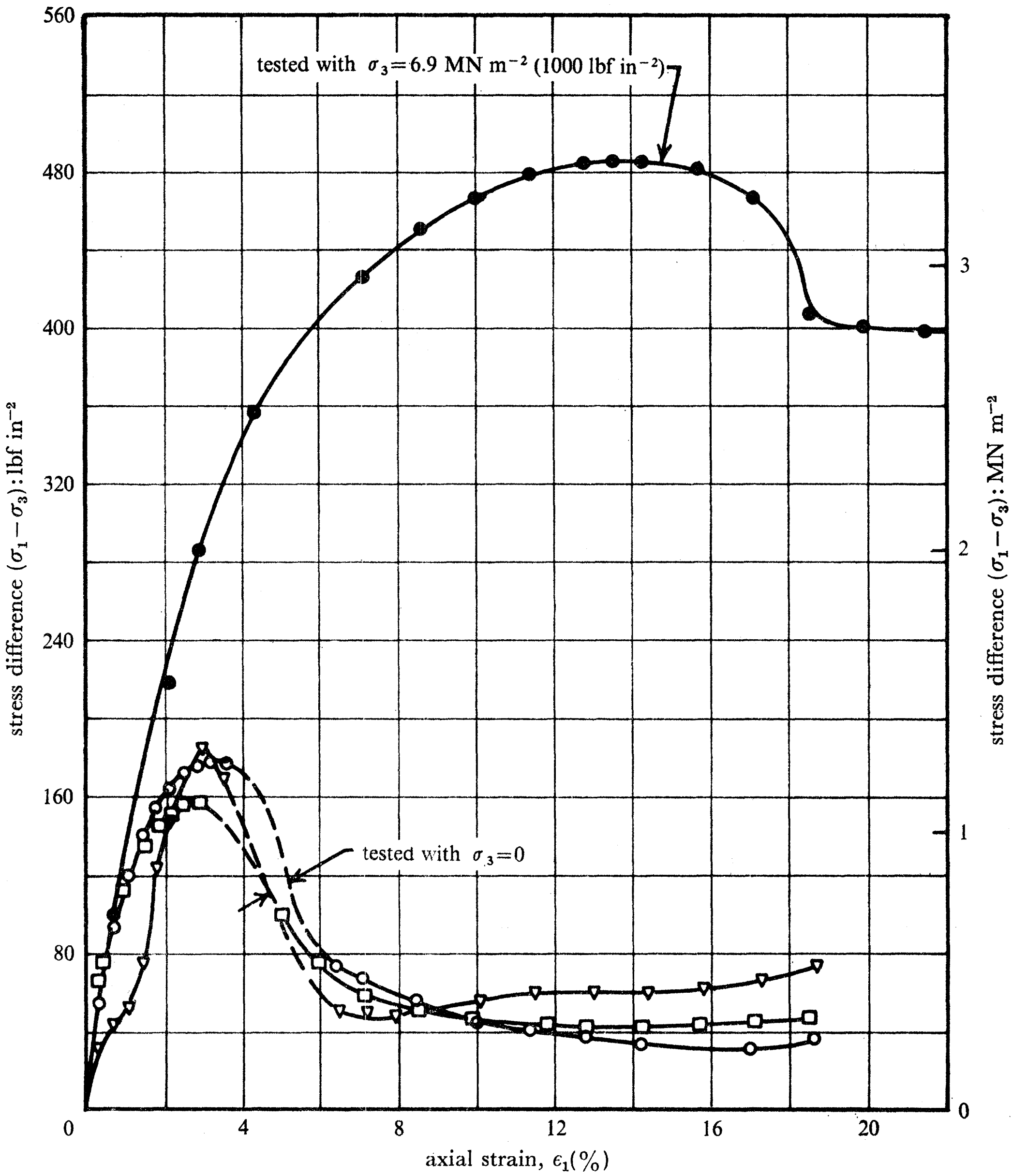

FIGURE 20. Comparison of confined and unconfined compression tests on samples of Kaolin consolidated with $\sigma_3 = 6.9$ MN m^{-2} (1000 lbf in^{-2}) and $u = 0$.

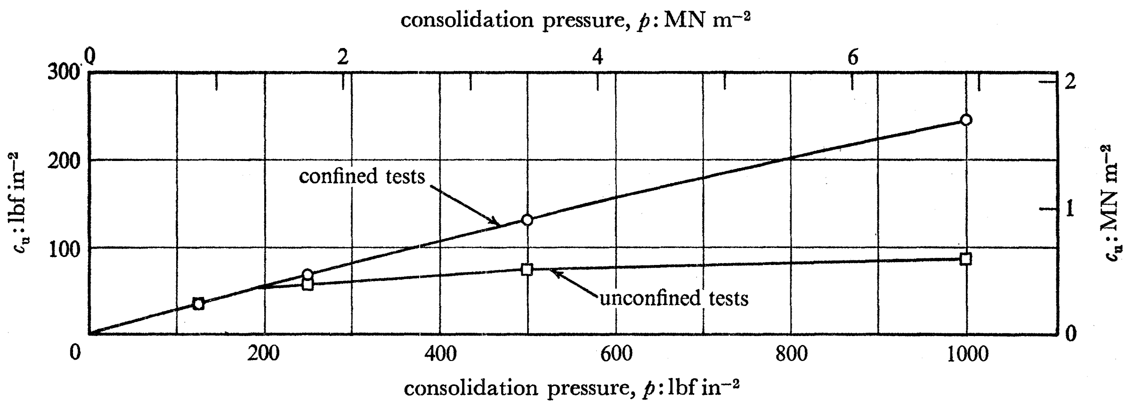

FIGURE 21. Relation between undrained strength c_u and consolidation pressure p for confined and unconfined tests on Kaolin. Where a number of tests were performed at a particular pressure, the average value is plotted.

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6. Discussion of results

The results of the tests carried out in stage 1 of the investigation demonstrate very clearly that reductions in confining pressure which do not result in negative pore-water pressures have only a very minor effect on the undrained strength of saturated samples of clay. In the series of tests described this amounted to a reduction of about 2 % in strength for a reduction in confining pressure of 3.45 MN m^{-2} (500 lbf in^{-2}).

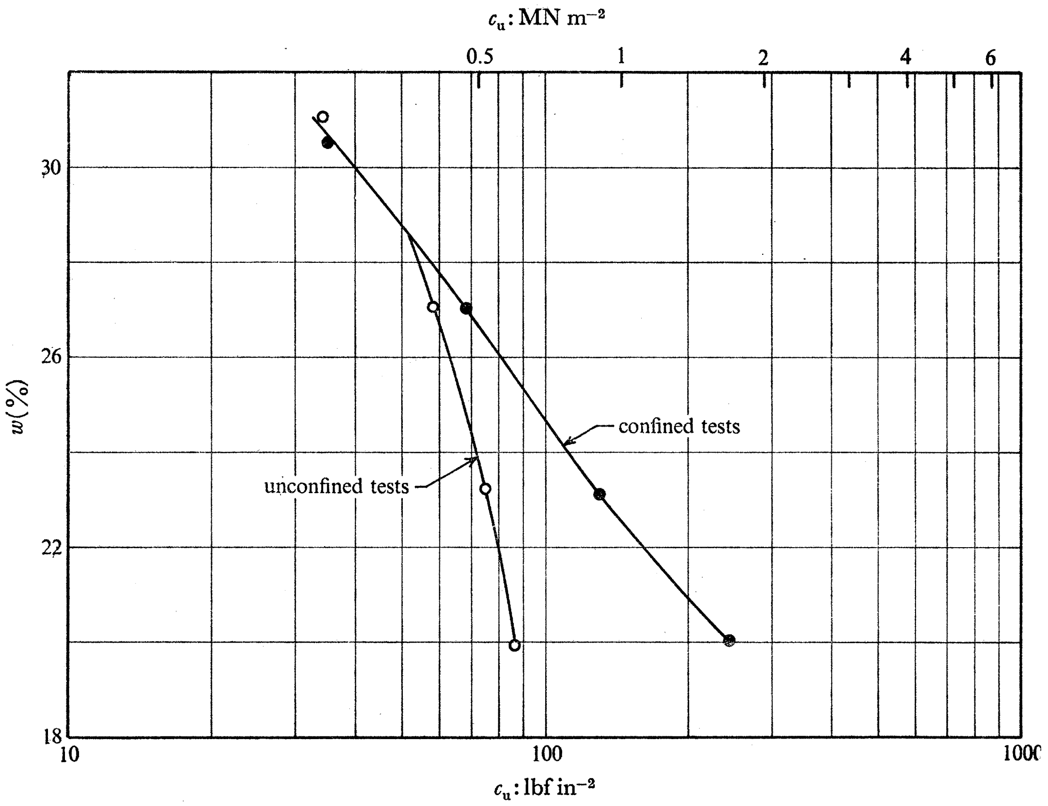

Figure 22. Relation between undrained strength c_u and water content w for both confined tests and unconfined tests on samples consolidated by external pressure: Kaolin.

The results of tests carried out under stage 2 of the investigation in which reductions in confining pressure led to pore-water tensions of various magnitudes showed equally clearly that a tensile component in the pore-water pressure is capable of replacing, with respect to its mechanical effects, an externally applied confining pressure up to a magnitude of almost 20.7 MN m^{-2} (3000 lbf in^{-2}) in the case of London Clay and about 1.38 MN m^{-2} (200 lbf in^{-2}) in the case of Kaolin.

At pore water tensions smaller than these limiting values the reduction in peak strength with confining pressure is a minor effect, of a similar order of magnitude to that noted above for saturated soils in the positive pore pressure range. However, there is a very significant change in the shape of the stress-strain curve after the peak (for example, London Clay with an inferred pore-water tension of 6.65 MN m^{-2} (965 lbf in^{-2}) before shear, figure 8). This indicates that high pore-water tensions result in greatly increased brittleness.

The loss in strength at larger cell pressure reductions indicates a failure of the pore water to sustain higher tensions without a departure from the condition of full saturation. As will be shown subsequently this is confirmed by the observed volume changes in a limited number of special tests.

In the case of the London Clay it could reasonably have been held that the pore water had failed in tension, since the inferred tensile stress in the pore water was above any value obtained by direct experimental measurement (see, for example, Temperley & Chambers 1946; Temperley 1946) although below the theoretical value of tensile strength.† However, the low value of breakdown pressure obtained for Kaolin, 1.38 MN m^{-2} (200 lbf in^{-2}), suggests that it is not a case of tensile failure in the pore water, but of the rupture of a meniscus in a capillary whose diameter is related to the effective pore size of the clay.

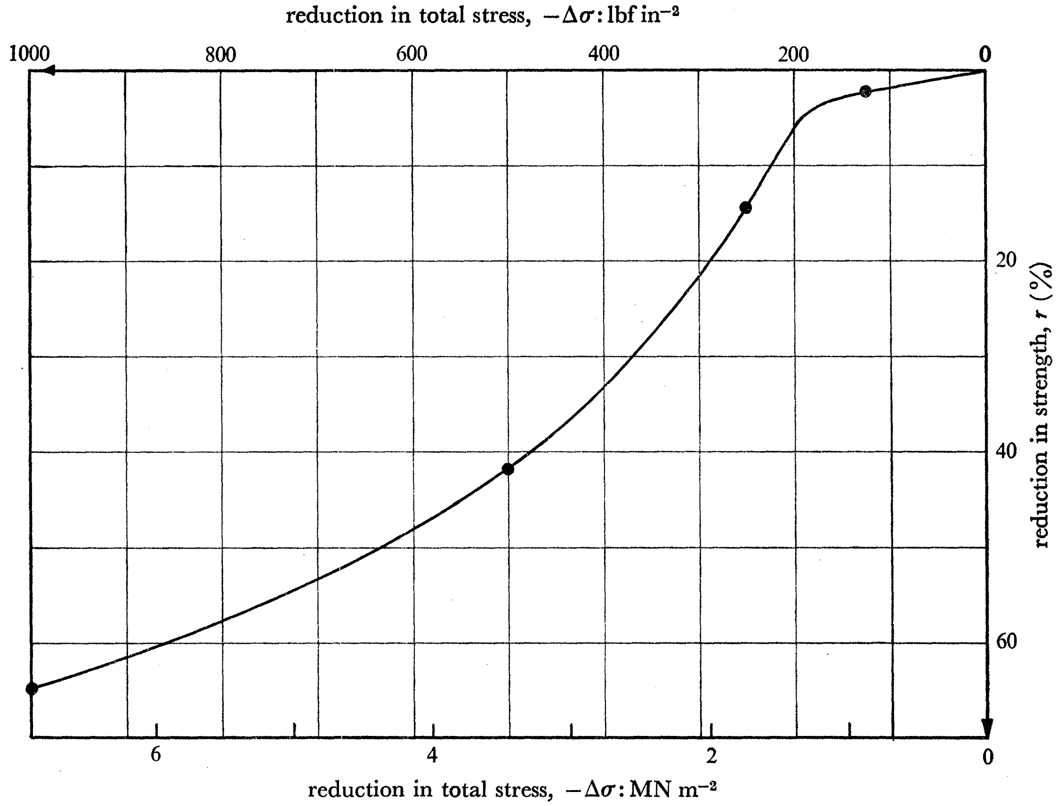

FIGURE 23. Relation between reduction in strength r and reduction in total stress $-\Delta\sigma$. Observed values from tests on Kaolin; initial pore-water pressure is 0.

The capillary rise h_c is given by the expression

$$h_c = \frac{2t_s}{R\gamma_w}\cos\alpha, \tag{18}$$

where γ_w denotes unit weight of water, t_s the surface tension, α the contact angle, and R the radius of capillary.

† Using Berthelot tubes, Temperley (1946) obtained values for the tensile strength of water ranging between 20 and 60 atm (1 atm ≈ 10^{5}N m^{-2}) and averaging 32 atm (470 lbf in^{-2}). His theoretical value is between 500 and 1000 atms. Green (1951), however, obtains a higher experimental value of 190 atm (2800 lbf in^{-2}).

The capillary pressure is therefore inversely proportional to the equivalent pore diameter, and the equivalent pore diameters for Kaolin and London Clay would therefore be expected to have the ratio of approximately 2820:200 or 14:1.

The equivalent pore diameters can be calculated independently from the permeabilities of the clays at the appropriate stress levels, the permeability K being calculated from the coefficient of consolidation c_v and the compressibility observed during the consolidation stage.†

Associating the value of permeability with the average effective stress level during consolidation we obtain values of the permeability K of 2.55–2.80×10^{-8} cm/s for Kaolin at a consolidation pressure of 1.38 MN m^{-2} (200 lbf in^{-2}) and 0.49×10^{-10} cm/s for London Clay at a consolidation pressure of 20.7 MN m^{-2} (3000 lbf in^{-2}).

Following Taylor (1948), the permeability K may be taken to be related to the equivalent pore diameter by the expression

$$K = \beta D^2 n, \tag{19}$$

where β is a constant which includes a shape factor, D denotes the equivalent pore diameter, and n denotes the porosity. Taking $n = 0.43$ for the Kaolin and 0.28 for the London Clay at the relevant pressures, and β to be the same for the two clays, we obtain the ratio of equivalent pore diameters as approximately 19:1 (in the range 18.4–19.3). This is in reasonably close agreement with the ratio of 14:1 given by the assumption that the breakdown pressure is controlled by capillarity. Indeed, very close agreement can hardly be expected, since

(1) the relation between equivalent pore diameter and permeability will be subject to the influence of surfaces forces and thus dependent on clay mineralogy and stress level, and

(2) for a given pore size distribution the 'equivalent diameter' controlling the two phenomena need not necessarily be the same.

It is of interest to note that the use of equation (18) leads to equivalent pore diameters of 2.06×10^{-1} μm for Kaolin and 1.46×10^{-2} μm for London Clay. The latter value is in the range of values obtained from shrinkage tests by Holmes (1955) who gives 2.3×10^{-2} μm and 0.9×10^{-2} μm for two clays with marked swelling characteristics and of similar clay mineralogy to London Clay (i.e. predominantly illite and kaolinite, with small amounts of montmorillonite; total clay fractions 64 % and 65 %).

In a sample subjected to a large enough reduction in total stress to cause partial saturation it is thus envisaged that passages filled with water vapour and gas (some air will have been in solution in the pore water in all tests) will propagate within the sample. In a clay with a relatively high expansibility there is no need to expect macroscopic migration of water relative to the sample boundaries and there is no significant experimental evidence of migration from the water content determinations. Whether the passages propagate from the external boundaries of the sample or from internal nuclei as well is difficult to determine, especially with small samples.

The total volume of the gas-filled voids can be calculated from the volume of the sample and its water content after the release of stress. As the samples subjected to compression tests fractured into a number of pieces accurate volume measurement was difficult. A sample subjected to the highest pressure in each series was therefore removed, without testing, for immediate volume

† Using the Terzaghi expression (Terzaghi 1943):

$$c_v = K/\gamma_w m_v,$$

where c_v denotes coefficient of consolidation and m_v denotes the compressibility and is replaced by C for equal all-round stress change.

determination, based on a series of very accurate height and diameter observations. A series of such observations was continued while the sample was allowed to dry slowly, and a final set of readings was taken after oven drying at 105 °C. A sample of each clay which had been stored after initial consolidation and swelling, but had not been heavily consolidated, was also subjected to a similar shrinkage test. The results of the four tests are presented in figures 24 and 25. The relations between dry unit weight γ_d (i.e., the weight of mineral material divided by the volume of the entire element), relative density of the particles G, void ratio e (i.e. volume of voids per unit volume of solids), water content w and degree of saturation S are given in standard textbooks on soil mechanics (see, for example, Lambe & Whitman 1969) as:

$$\gamma_d = \frac{G\gamma_w}{1+wG/S}, \tag{20}$$

$$\gamma_d = \frac{G\gamma_w}{1+e}, \tag{21}$$

$$Gw = Se, \tag{22}$$

where γ_w is the unit weight of water at the relevant temperature.

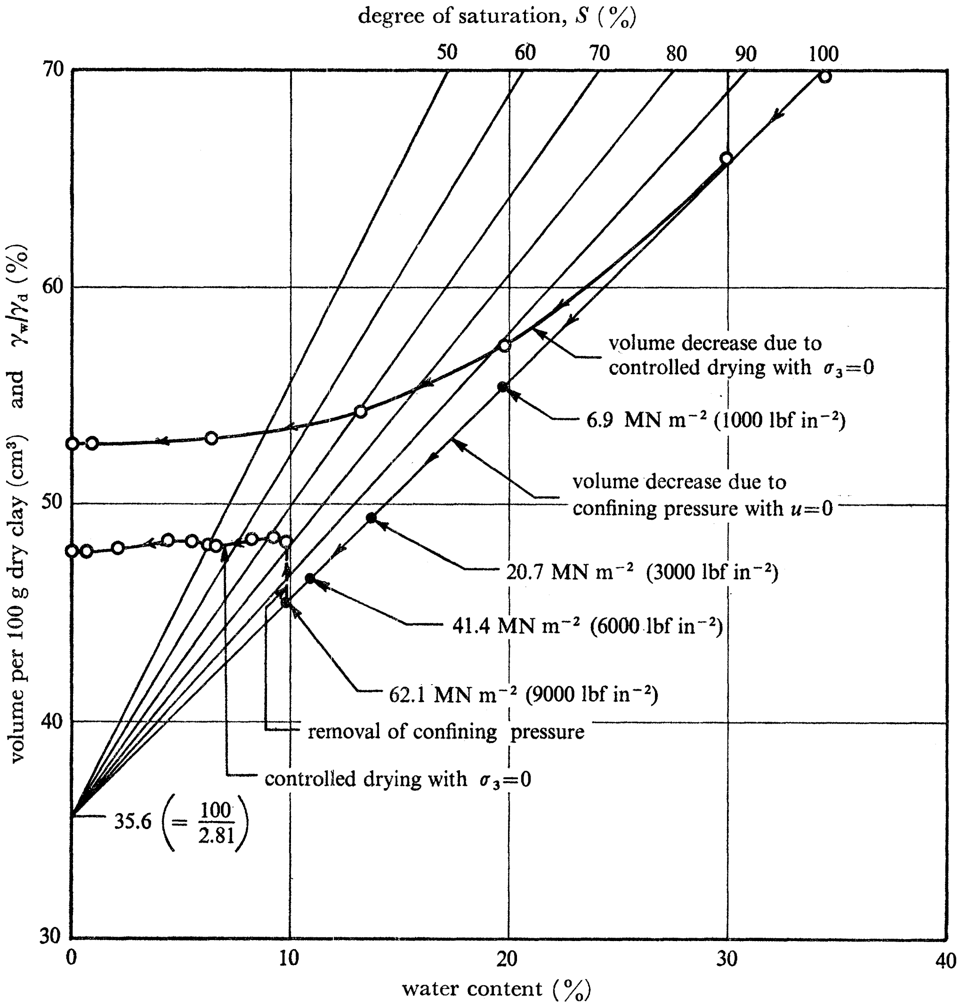

FIGURE 24. Relations between volume change and water content during consolidation, undrained stress release and drying: London Clay; relative density G assumed constant at 2.81 throughout pressure range.

Using these expressions, we can obtain the void ratio of the saturated sample at the end of consolidation from the final water content, and the void ratio after release of the cell pressure from the dry unit weight on removal from the cell.

These calculations show that on the release of cell pressure the sample of London Clay consolidated at 62.1 MN m^{-2} (9000 lbf in^{-2}), expanded 6.0 %, and that the percentage air voids was 5.7 % and the degree of saturation was 78 %. For Kaolin consolidated at 6.9 MN m^{-2} (1000 lbf in^{-2}), the corresponding values are: expansion 12.7 %, air voids 11.3 % and degree of saturation 73 %. It is of interest to note that the one dimensional consolidation tests of Westman (1932) lead to comparable values, Georgia Kaolin consolidated to 57.1 MN m^{-2} (8280 lbf in^{-2}) giving an expansion of 11.6 %, air voids of 10.4 % and a degree of saturation of 66 %.

The determination of degree of saturation and percentage air voids are subject to a small error due to the variation of γ_w with tensile stress in the water. The magnitude of this error is examined in appendix 2.

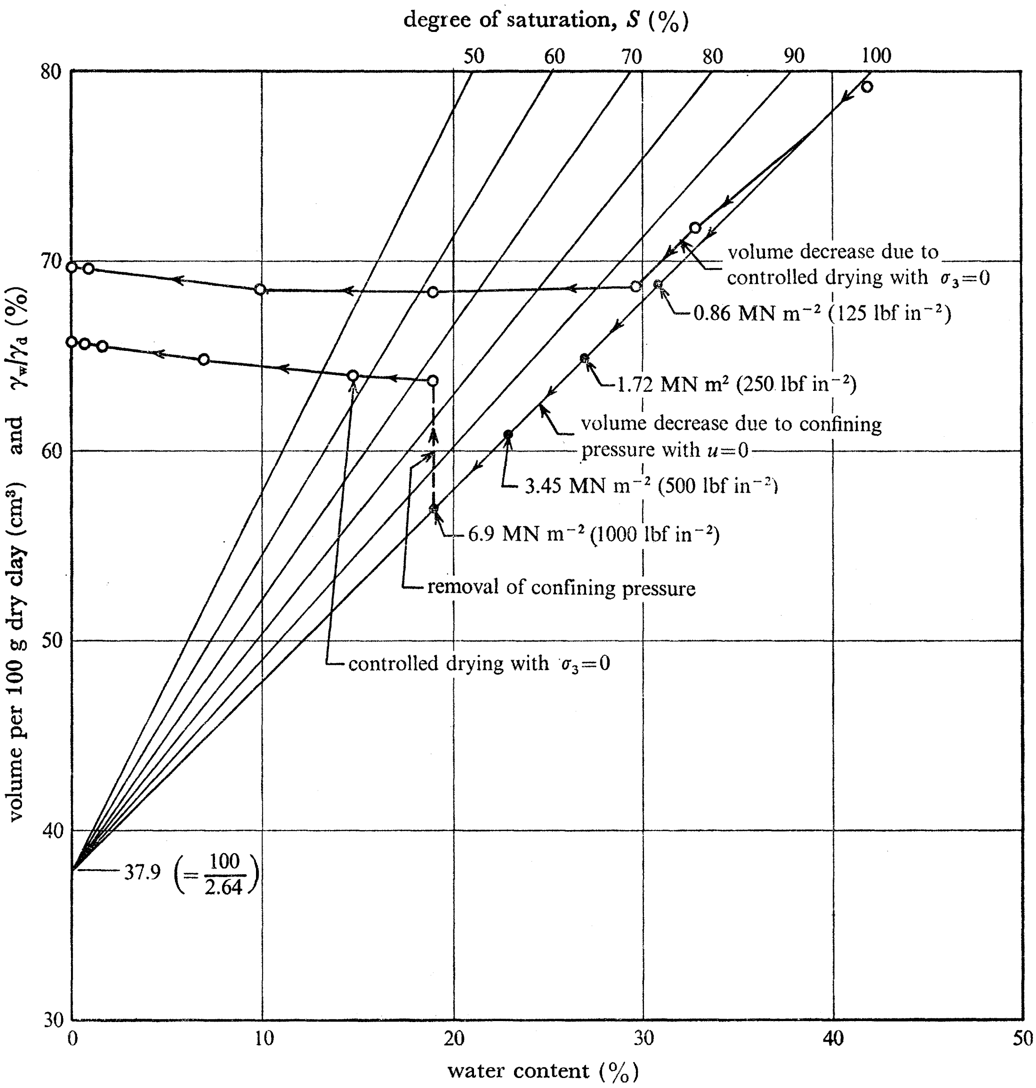

FIGURE 25. Relations between volume change and water content during consolidation, undrained stress release and drying: Kaolin; relative density G assumed constant at 2.64 throughout pressure range.

After removal from the triaxial cell the samples released from these stresses showed no significant tendency to change in volume with time (until drying took place). This suggests that, once initiated, the propagation of cavitation within the pore water is fairly rapid and complete.

On drying, the London Clay sample underwent a small decrease in volume (figure 24) of about 1 % which occurred mainly on the removal of the last 5 % of water content. In contrast the Kaolin (figure 25) showed an expansion of 3 % on drying. These results are consistent with current view of the role of water layers in soil mechanics as illustrated, for example, by the description given by Lambe & Whitman (1969) of the transmission of force through a flocculated soil. This view was forseen by Westman (1932) who considered that, at least at higher pressures, 'the data obtained could be readily explained by specifying a system of flexible, mechanically weak, solid particles in solid contact with each other'.

The primary purpose of this investigation was to study the influence of pore-water tensions on the undrained strength of clay. So far we have shown that up to a limiting pore-water tension the sign of the pore-water pressure has little influence on the strength, but a marked influence on the shape of the post-peak section of the stress–strain curve. The magnitude of the limiting pore-water tension varies radically with clay type and correlates with equivalent pore diameter in a manner similar to capillary tension. Three further aspects of the influence of pore-water tension on strength deserve a brief comment:

(*a*) the rate of increase of unconfined strength with consolidation pressure above the limiting pressure at which partial saturation occurs in the unconfined sample,

(*b*) the increased brittleness associated with pore-water tensions, and

(*c*) the consolidation of samples by the pore-water tension associated with drying.

(*a*) *The rate of increase of unconfined strength with consolidation pressure above the limiting pressure at which partial saturation occurs*

This is difficult to predict theoretically since the substantial volume increase occurring when the limiting pore-water tension is exceeded results in the sample behaving as both an over-consolidated as well as a partly saturated soil. However, several correlations of interest may be noted.

Normally consolidated and lightly over-consolidated remoulded samples of London Clay show an almost unique relationship between undrained strength and water content and hence, being fully saturated, with void ratio (see Table I–IV of Bishop (1971)). In table 6, undrained strengths are given for saturated samples tested at the void ratios observed on removal of the confining pressure for both London Clay and Kaolin (the maximum pressure in each series). For London Clay this represents an underestimate of the unconfined strength at this void ratio, though for Kaolin the agreement is close.

This difference reflects the relative importance of the contribution made by true cohesion (as defined by Hvorslev 1937) to the strength of the two clays. In table 6, values are given of Hvorslev's parameter κ (the ratio of true cohesion to consolidation pressure) taken from typical values given by Gibson (1953), together with values of the *activity* of the clay as defined by Skempton (1953).

It will be seen from table 6 that the change in the unconfined strength with consolidation pressure $\mathrm{d}c_u/\mathrm{d}p$ is almost linearly related to the value of *activity* of the clay, if based on average values over the full range of partial saturation observed. The flattening of the curve for Kaolin (figure 21) suggests a better correlation with κ at higher pressures, but this is subject to confirmation by the measurement of values of this parameter in the relevant stress range.

A correlation of the unconfined strength of the samples consolidated in this range with cohesion is also consistent with the work of Schmertmann & Osterberg (1960) who showed that the cohesion term reached a peak very early in the test, while the friction term required 10–20 times the strain for its full mobilization to be approached. It will be noted that both for London Clay (figures 10 and 11) and Kaolin (figures 19 and 20) the failure strains of the unconfined samples in which full saturation is not maintained drop to 35–45 % (London Clay) and 20–25 % (Kaolin) of the failure strains of the samples tested in the positive pore pressure range. This implies that the friction contribution is reduced in the unconfined tests in which full saturation is not maintained, especially in the case of Kaolin, not only due to the reduction in effective stress but also due to the reduction in failure strain.

Table 6. Relations between the increase in unconfined strength above the limiting pressure, the activity of the clay, and Hvorslev's cohesion parameter

material	consolidation pressure		unconfined value of c_u		strength of saturated sample at same void ratio		$\frac{dc_u}{dp}$	activity†	Hvorslev κ
	MN m^{-2}	lbf in^{-2}	MN m^{-2}	lbf in^{-2}	MN m^{-2}	lbf in^{-2}			
London Clay	20.7	3000	4.04	586	—	—	0.069	0.91	0.10‡
	62.1	9000	6.89	1000	5.24	760			
Kaolin	1.72	250	0.40	58.2	—	—	0.037	0.44	0.02‡
	6.9	1000	0.60	86.3	0.59	85.0			

† Plasticity index/percentage clay.
‡ Approximate, based on typical values given by Gibson (1953) for the low pressure range.

Table 7. Loss in strength for 2 % post-peak strain for confined and unconfined tests

material	consolidation pressure		loss in peak strength when tested unconfined (%)	confined test – loss in strength for 2 % post peak strain (%)	unconfined test – loss in strength for 2 % post peak strain (%)
	MN m^{-2}	lbf in^{-2}			
London Clay	6.9	1000	0.5(2)†	1.3	3.2
	20.7	3000	6	0.9	63
	41.4	6000	38	0.4	88
	62.1	9000	42	0.6	87
Kaolin	0.86	125	3	8.6	7.5
	1.72	250	11	6.0	6.6
	3.45	500	42	7.1	63
	6.89	1000	65	1.4	42

† The value of 2 % is obtained if the fifth test, which differs substantially from the average, is included.

(b) *The increased brittleness associated with pore-water tensions*

One of the most noteworthy features of the tests on both London Clay and Kaolin is the change in the shape of the post-peak section of the stress–strain curve with increasing pore-water tension, in particular when this exceeds the limit at which full saturation can be maintained.

Even in the range of positive pore-water pressure, saturated samples of both London Clay and Kaolin are strain softening, the brittleness index I_B† being estimated to be about 70 % for

† The brittleness index I_B for undrained tests is defined by (Bishop 1971) as

$$I_B = [(c_u)_f - (c_u)_r]/(c_u)_f,$$

where the suffices f and r denote failure (peak) and residual states respectively.

normally consolidated and lightly overconsolidated remoulded London Clay (Bishop 1971, Table I–VII). However, the rate of decrease in strength observed at the strains achieved in the triaxial test is relatively small (figures 8–11 and 17–20). In contrast, the unconfined samples having the higher initial consolidation pressures show a sudden loss of strength occurring mainly within an additional 1–2 % of axial strain (figures 10, 11, 19 and 20, and table 7). It is apparent from table 7 that this abrupt drop in strength is more marked in the London Clay than in the Kaolin. In the London Clay it amounts to 63 % for a cell pressure reduction of 20.7 MN m^{-2} (3000 lbf in^{-2}). In the Kaolin the drop in strength does not rise to 63 % until the limiting pressure (*ca.* 1.38 MN m^{-2} (200 lbf in^{-2})) has been exceeded by more than 100 %. The actual value of the brittleness index I_B is not readily ascertained from triaxial tests due to the limited strains imposed. In addition the top of the sample in the present apparatus is constrained by friction from moving freely sideways to accommodate the kinematics of a large displacement on a single rupture surface. The highest value of I_B observed was for London Clay at the maximum consolidation pressure (figure 11), which gave a value of 92.5 %. The absence of cohesion across the rupture surface when the sample was removed from the rubber membrane suggests that the true value of I_B was very close to 100 % for this sample and for all samples with high pore-water tensions.

(*c*) *The consolidation of samples by the pore-water tension associated with drying*

A series of unconfined compression tests was carried out on samples of the London Clay consolidated by slow shrinkage from the same initial water content as those consolidated by a confining pressure. The strength–water content relations are plotted in figure 14.

It will be noted that the strengths are considerably lower, for a given water content, than those of samples consolidated by a confining pressure, and tested unconfined. From the curve representing the strength–water content relation it can be seen that at a water content of 19.7 % (corresponding to a consolidation pressure of 6.9 MN m^{-2} (1000 lbf in^{-2})), the strength of a sample consolidated by shrinkage is 75 % of that consolidated by a confining pressure.

At 13.6 % water content (corresponding to 20.7 MN m^{-2} (3000 lbf in^{-2})) the strength is 61 % of that of the sample consolidated by a confining pressure. At a water content of 10.2 % (corresponding to a consolidation pressure of 62.1 MN m^{-2} (9000 lbf in^{-2})), the strength is 59 % of that of a sample consolidated by a confining pressure and tested unconfined, and only 33 % of that of a sample tested under a confining pressure equal to the consolidation pressure.

It is thus apparent that there is no unique relation between strength and water content, even for the unconfined samples at water contents corresponding to consolidation pressures less than the value leading to partial saturation in unconfined specimens.

An explanation for this difference may be found in the volume-change water-content relation for the shrinkage test on the London Clay plotted in figure 24. The equation for the saturation lines on this plot may be obtained by re-arranging equation (20)

$$\frac{\gamma_w}{\gamma_d} = \frac{1}{G} + \frac{w}{S}. \tag{23}$$

It will be seen that the degree of saturation for a sample following the shrinkage curve is 92 % at 19.7 % water content, 73 % at 13.6 % water content and 57 % at 10.2 % water content. These values may be compared with 100, 98 and 78 % respectively for the samples consolidated under a confining pressure and released (the first and second values being estimated from strength changes, the third being a measured value). The void ratio of the sample following the shrinkage curve is thus higher, at each reference water content, than that of a sample consolidated by an applied pressure and then tested unconfined.

It is of interest to note that a plot of strength against void ratio at the beginning of the compression test (figure 26) indicates that for London Clay no unique relation exists on this basis either and that uniqueness is unlikely to be achieved even after correcting for the volume change

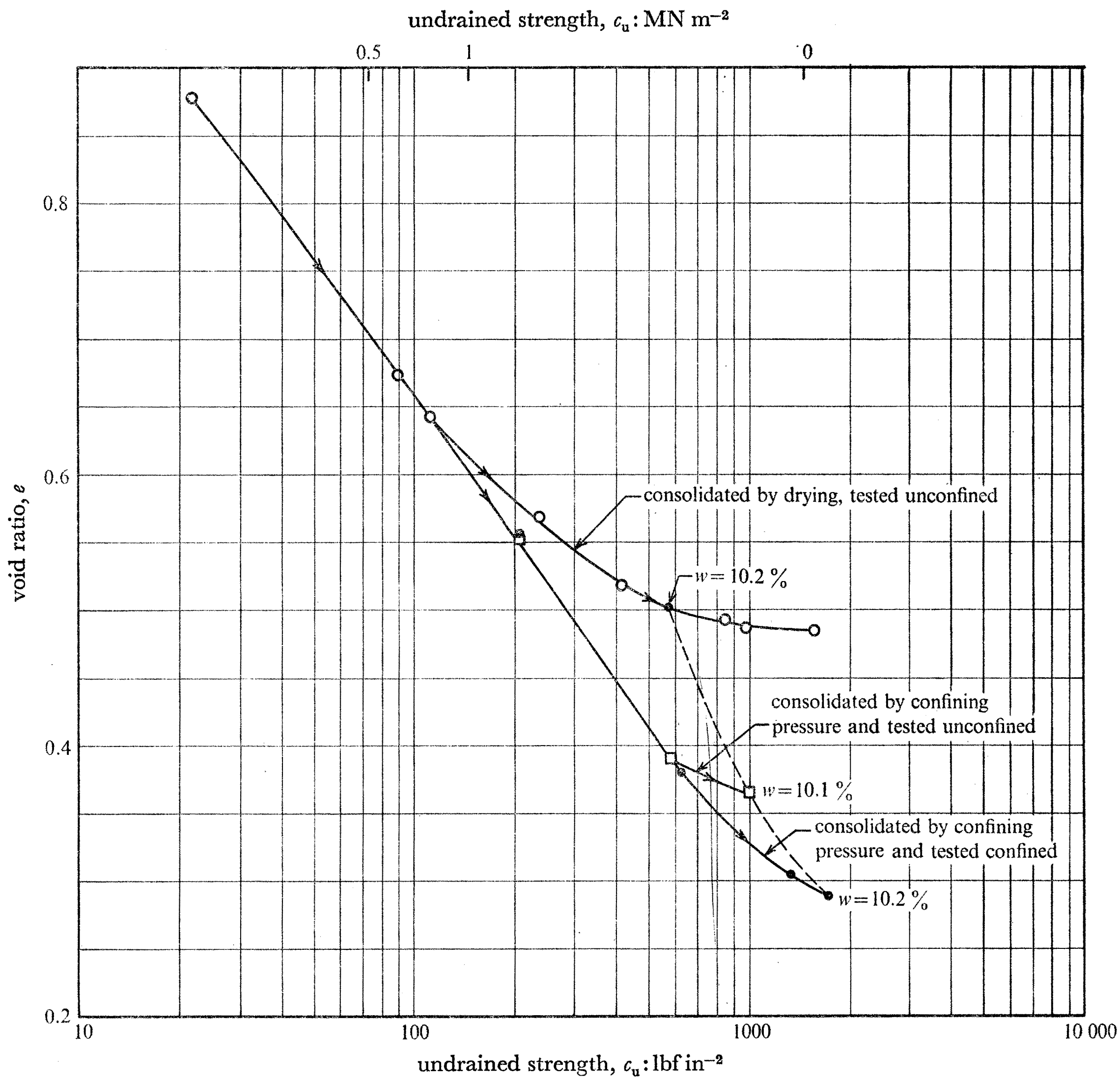

FIGURE 26. Relations between undrained strength c_u and void ratio e at the beginning of the compression test for confined tests, unconfined tests on samples consolidated by external pressure and unconfined tests on samples consolidated by drying: London Clay.

preceding failure in compression. In the limit a sample on the drying curve has a strength of 10.7 MN m^{-2} (1557 lbf in^{-2}) for an initial void ratio of 0.484 and zero water content, while the confined test at the highest consolidation pressure gives almost the same strength (11.9 MN m^{-2} (1723 lbf in^{-2})) for an initial void ratio of 0.287 and a water content of 10.2 % (figure 26). Even with the assumption of a Poisson ratio of 0.2 for the dry sample the void ratio at failure drops only to 0.407.

A discussion of the high pore-water tensions associated with drying in the range of water contents under consideration and of the modified effective stress equation necessary to relate these pore-water tensions to the strength of partly saturated soils is outside the scope of this paper. Reference may be made to Holmes (1955), Bishop (1959), Croney & Coleman (1960), Aitchison (1960), Bishop *et al.* (1960) and Bishop & Blight (1963).

The only directly comparable test data are given by Croney & Coleman (1960) who carried out standard unconfined compression tests on samples of undisturbed London Clay dried to known initial suctions in the pressure-membrane and vacuum-desiccator apparatus. The maximum value of c_u observed was about 3.79 MN m^{-2} (550 lbf in^{-2}) at an inferred suction of 689 MN m^{-2} (100000 lbf in^{-2}) and almost zero water content. This value of c_u is only 35 % of the value obtained in the present series of tests on remoulded clay of similar index properties, and reflects the non-uniformity of particle size distribution in a natural soil, together with the effects of specimen size on strength in an undisturbed soil with a fissured structure. This latter effect could account for a reduction of almost 50 % in undrained strength in the relevant range of sample sizes in the weathered zone of the London Clay (Bishop 1971, Fig 1–29).

7. Conclusions

(1) For saturated samples of clay the influence on undrained strength of large reductions in confining pressure which do not result in negative pore-water pressures is very small. This is in general agreement with predictions based on the relative expansibilities of the soil structure and of the pore water.

(2) The influence on undrained strength of large reductions in confining pressure which do result in pore-water tensions is likewise very small until a limiting tension is reached above which a departure from full saturation occurs. The value of this tension is about 19.4 MN m^{-2} (2820 lbf in^{-2}) (i.e., 0.94×3000) for the London Clay and about 1.38 MN m^{-2} (200 lbf in^{-2}) for the Kaolin. This value correlates with equivalent pore diameter in the same way as the limiting capillary tension.

Larger reductions in confining pressure lead initially to rapidly increasing reductions in undrained strength which reach 42 % for the London Clay and 65 % for the Kaolin with the ranges of pressure investigated. It should be observed that the limiting pore-water tension and the percentage reduction in strength for a given reduction in total stress are not unique soil characteristics, but will depend on stress history. For example, reductions in confining pressure of 20.7 and 41.4 MN m^{-2} (3000 and 6000 lbf in^{-2}) will not lead to the same loss in strength when applied to a sample consolidated to 62.1 MN m^{-2} (9000 lbf in^{-2}) as when applied (as in the present tests) to samples consolidated only to 20.7 and 41.4 MN m^{-2} respectively. This is due to initial differences in expansibility and in equivalent pore diameter.

(3) The gain in unconfined strength with consolidation pressure above the limiting pressure correlates both with the activity of the clay and with the Hvorslev cohesion component.

(4) Large pore-water tensions are associated with a very marked change in shape of the post-peak section of the stress–strain curve, the unconfined samples of London Clay in particular showing a very brittle behaviour and negligible cohesion after rupture. This clearly indicates that undrained residual strength is a function of total normal stress, as suggested by Bishop (1971).

(5) Samples consolidated by drying do not show the same strength–water content or strength-void ratio relations as those consolidated by applied pressure at the relatively high pore-water

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tensions under consideration. This appears to be due, in part at least, to a departure from full saturation at a much higher water content and thus at a smaller pore-water tension than that inferred for samples consolidated by an applied pressure and tested unconfined. It may to a smaller extent reflect the influence of residual stresses resulting from pore-pressure gradients during drying.

(6) The test results have important implications with regard to the stress release which inevitably occurs on the taking of samples from deep boreholes or shafts, to the choice of laboratory test conditions, and to the field behaviour of clay strata subject to undrained stress reduction associated with deep cuts and excavations, seismic disturbance or shock-waves.

Since the state of stress in the ground does not in general approximate to an equal all-round pressure, the pore-water tension in an ideal undisturbed and fully saturated sample on stress release is not equal to the effective vertical stress in the ground before sampling even for the case of $B = 1$. In normally consolidated clay strata the coefficient of earth pressure at rest K_0 (the ratio of the horizontal effective stress to the vertical effective stress) lies in the range 0.4 to 0.7 depending on the plasticity index, on the basis both of laboratory tests (Bishop 1958; Simons 1958) and field data (Bjerrum & Anderson 1972). On the basis of elastic theory the ratio of the pore-water tension $-u_s$ to the vertical effective stress σ'_v would be $\frac{1}{3}(1+2K_0)$ for $B = 1$. Values of $-u_s/\sigma'_v$ in the range 0.6–0.8 would thus be expected for perfect sampling.

However, normally consolidated soil does not behave as an ideal elastic material, and pore pressure changes measured in the laboratory on samples subject to the stress changes occurring during perfect sampling (Bishop & Henkel 1953; Skempton & Sowa 1963; Ladd & Lambe 1963) give lower values particularly in soils of low plasticity. On the basis of these results values of $-u_s/\sigma'_v$ in the range 0.35–0.75 might be expected in real soils.

However, mechanical disturbance during the sampling operation can account for a far greater reduction in the ratio $-u_s/\sigma'_v$ in the case of normally consolidated soils. Laboratory tests (Bishop & Henkel 1953) show that a single application and removal of a shear stress equal to the strength may reduce this value by more than 50%, while complete remoulding may reduce it by more than an order of magnitude (Croney & Coleman 1960; Bishop 1960; Skempton & Sowa 1963). Field data presented by Ladd & Lambe (1963) suggests that a drop of 80% below the value for 'perfect sampling' is typical for current sampling procedures in normally and lightly overconsolidated clays.

In contrast heavily overconsolidated soils have values of K_0 in excess of unity, show a behaviour which approximates more closely to the ideal elastic assumption and do not show a marked drop in pore-water tension on remoulding. In fact in some heavily over consolidated clays a single application and removal of a shear stress may increase the residual pore-water tension (Bishop & Henkel 1953) as also will complete remoulding (Croney & Coleman 1960). The cutting of a smaller sample from a large block sample consolidated under a known equal all-round stress resulted in a small but not significant reduction in the residual pore-water tension in the case of London Clay (Skinner, unpublished data).

Field data for over-consolidated clay thus show very high ratios of $-u_s/\sigma'_v$, varying with depth from 2.3 to 1.3 for the London Clay east of London (Skempton 1961) and from 2.7 to 1.7 for the London Clay west of London (Bishop *et al.* 1965). Values of K_0 calculated from these ratios lie in the ranges 2.8–1.5 and 3.4–1.7 respectively. Higher values of $-u_s/\sigma'_v$ have been reported by Blight (1967) for an expansive clay.

The largest value of residual pore-water tension reported from any of these sites is -0.76 MN

m^{-2} (110 lbf in^{-2})† for west London at a depth of 42 m (138 ft). For this value to be achieved in a *normally consolidated* sample of medium plasticity it would have to be recovered from a depth of some 1000 m on the assumption that the initial pore pressure distribution is hydrostatic.

The maintenance of full saturation under the residual pore-water tensions encountered in normal sampling operations should thus present no difficulty in homogeneous clays of pore size similar to the London Clay or even the Kaolin used in the present series of tests. However, the pore water in any macro-voids or segregated zones of coarser particles such as varves and silty partings is likely to cavitate and drain into the adjacent clay. Likewise continuity of the pore water will be lost across fissures, joints and bedding planes. This may result in a complete loss in strength, or in a substantial reduction in the unconfined strength (see, for example Bishop & Henkel 1962, Fig. 66).

The reduction in pore-water tension in the more disturbed zones of a borehole sample will result in migration of pore water, at constant overall volume, to the least disturbed zones and thus to a general reduction in strength.

The mitigation of these effects in the laboratory by the re-application of the estimated *in situ* total stresses without drainage or with drainage to an appropriate back-pressure is outside the scope of the present paper, as are such factors in the sampling procedure as the choice of sample size to minimize the proportion of very disturbed material and the limitation of access to free water particularly in overconsolidated clays.

The two factors of most significance in the field behaviour of clay masses subject to undrained stress release are the increased brittleness and the loss in strength when the pore-water tension results in loss of continuity of the pore water in silty partings, fissures and joints.

The authors are indebted in particular to Dr A. E. Skinner, who has been associated with the senior author for a number of years in the development of high pressure triaxial equipment. Dr Skinner has been largely responsible for putting the idea of a hydraulically balanced ram into practice and designed the internal load transducer used in this investigation.

Mr L. D. Wesley carried out a very careful series of relative density determinations on the two clays.

Mr D. Evans has given valuable assistance in the laboratory and Mr E. Harris has prepared the illustrations.

Appendix 1

Two novel features incorporated in the high pressure triaxial cell are the hydraulically balanced loading ram and the internal load transducer:

(*a*) *The hydraulically balanced ram*

The central section of the loading ram (figure A 1.1) is enlarged to form a piston having a diameter $\sqrt{2}$ times that of the ram, sliding with an appropriate oil seal in a cylinder formed in the head of the triaxial cell. Oil at cell pressure communicates through passages drilled in the ram with the annular upper surface of the piston. The cylinder below the piston is bled to atmospheric pressure.

If σ_3 is the fluid pressure in the cell, and d the diameter of the ram, then the upthrust on the

† Pore-water tensions of this magnitude cannot be measured directly and are obtained by a number of indirect methods discussed in the papers referred to.

ram due to the cell pressure is $\frac{1}{4}\pi d^2\sigma_3$. The downthrust on the piston is $\frac{1}{4}\pi[(\sqrt{2}\,d)^2-d^2]\,\sigma_3$ and thus also equals $\frac{1}{4}\pi d^2\sigma_3$. However high the cell pressure, the axial load applied to the ram is equal only to the compression strength of the sample together with the ram friction.

A further advantage is that there is no change in the total volume of oil in the triaxial cell and loading head as the ram enters the cell. Thus the use of a high strain rate does not lead to a build-up of pressure in the cell due to hydraulic resistance in the small bore pressure tubing or to the characteristics of the pressure control system.

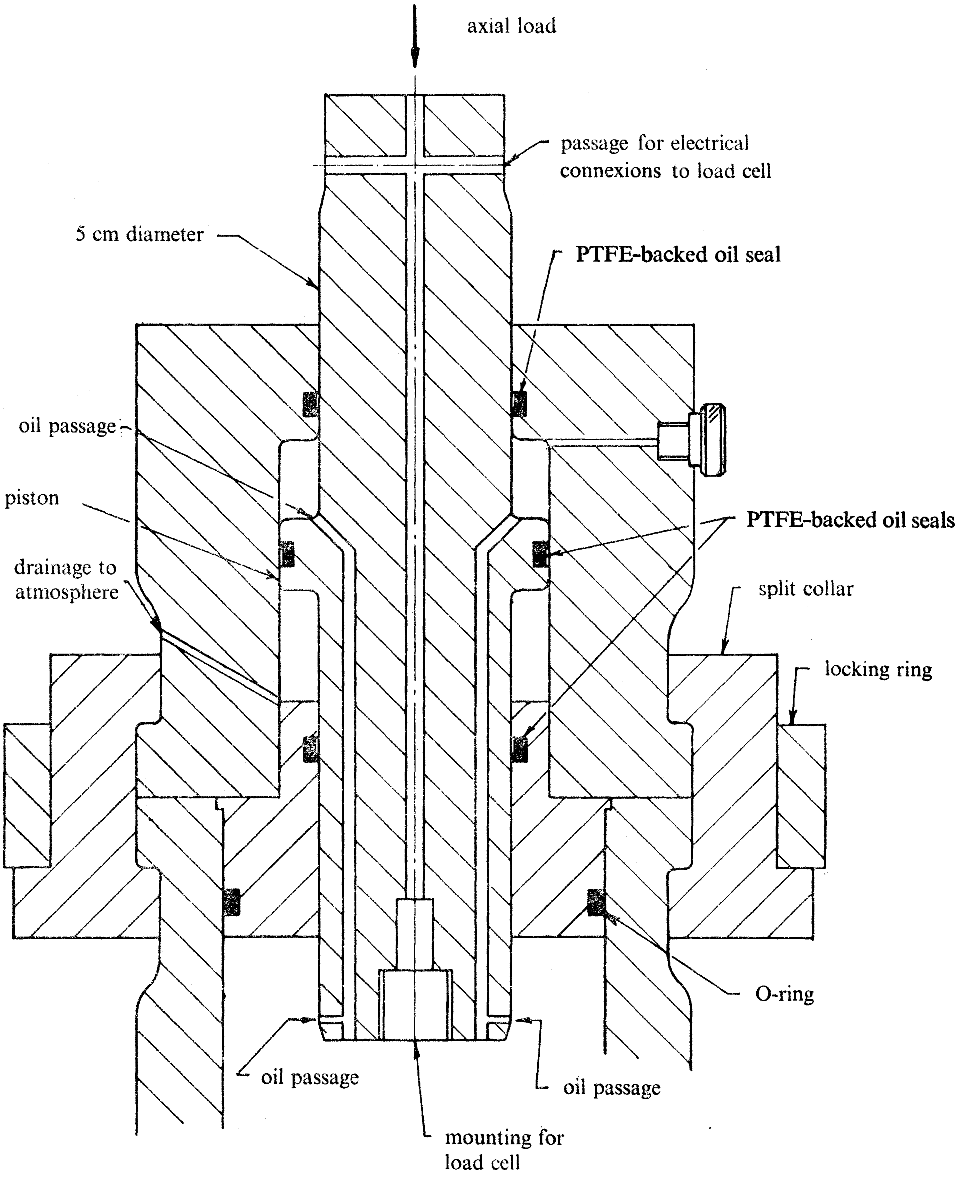

FIGURE A1.1. Head of triaxial cell with hydraulically balanced ram.

(b) *The internal load transducer*

The load transducer consists, in effect, of three triangular cantilevers of uniform thickness radiating from a common boss and bearing on the edge of a groove in the cylindrical loading cap (figure A1.2). As the bending moment and moment of resistance both increase linearly from the apex of each triangular cantilever the surface strains are almost uniform, and the calibration is therefore insensitive to the location of the electrical resistance strain gauges.

The cylindrical loading cap is filled with oil and sealed with a flexible nitrile rubber cover. A change in cell pressure thus has no significant effect either on the zero or on the calibration. The resistance gauges are wired to give complete insensitivity to eccentricity of loading and to the horizontal component of load.

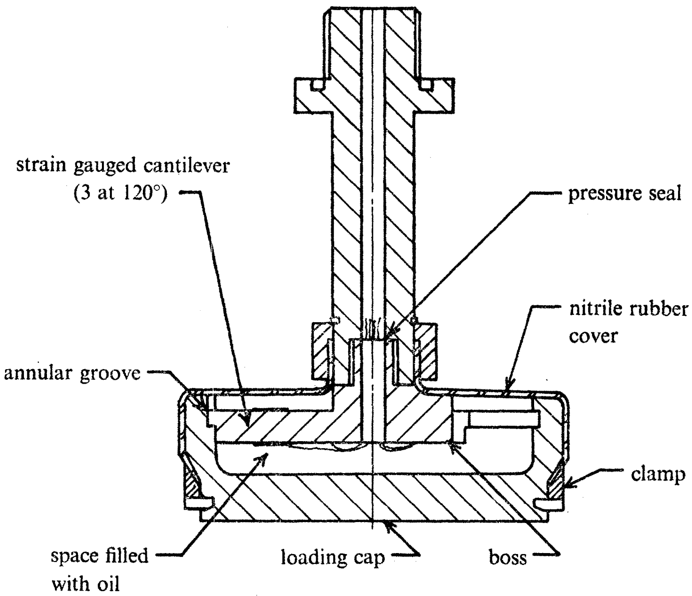

FIGURE A1.2. Internal load transducer.

APPENDIX 2

Equations (20)–(22) are based on the assumption that the unit weight of water in the voids of the soil is equal to the value of the unit weight of water used as the reference value for the determination of the relative density G. This should be the unit weight of water at 4 °C. This point was appreciated by Taylor (1948), but is not referred to in the description of the standard method of measurement of specific gravity in B.S. 1377 (1967). The difference due to temperature is small but just significant at the usual standard laboratory temperature of 20 °C, when the unit weight of water is 0.9982 gf/ml.

The difference due to the compressibility (or expansibility) of water is substantially greater at high pore-water pressures (or tensions), and is significant at the tensions induced in the tests on London Clay by stress release or drying. On the assumption that the coefficient C_w has the same value in tension as in compression (an assumption made by Temperley & Chambers 1946) the unit weight of water would decrease by 1.01 % at a tension of 20.7 MN m^{-2} (3000 lbf in^{-2}) and by 3.03 % at 62.1 MN m^{-2} (9000 lbf in^{-2}).

The pore-water tension in the samples in which partial saturation has occurred cannot be estimated with accuracy, but will clearly be in excess of the breakdown value (19.4 MN m^{-2} (2820 lbf in^{-2}) for London Clay) and below the value calculated for the particular consolidation pressure on the assumption of full saturation. This gives values of the unit weight of water ranging from 0.9888 to 0.9750 gf/ml for the highest consolidation pressure. These values imply possible errors in the calculated degree of saturation in the range 1–2½ %, and in the calculated percentage air voids of 4–9 % for the sample at the highest consolidation pressure.

The expansion which would have occurred on stress release if there had been no departure from full saturation can be calculated from the expression for undrained compressibility $\bar{C}$ given by Bishop (1966, 1973)

$$\bar{C} = \frac{nC_w + (1-n)\,C_s - C_s^2/C}{1 + nC_w/C - (1+n)\,C_s/C}. \tag{A 2.1}$$

For the sample consolidated under 62.1 MN m^{-2} (9000 lbf in^{-2}) this gives a value for the expansion of 0.60 %.

As this expression for undrained compressibility is based on the same physical assumptions as the pore pressure equation (equation (2)), in particular that all the pore water has the compressibility of bulk water, it is of interest to check its accuracy. The present triaxial cell was not designed to measure undrained compressibility, but on the assumption of isotropy under cyclical applications of equal all-round pressure the value of $\bar{C}$ can be calculated from the changes in height of the specimen. The observations were complicated by a time-dependent interchange of water between the sample and the rubber membrane enclosing it as the pore-water tension is changed. Corrected 'instantaneous' values for two different porosities do not suggest any substantial error in equation (A 2.1).

A further assumption in the use of equations (20)–(22) to determine final void ratio under stress and degree of saturation on undrained stress release is that the value of the relative density G of the clay particles remains unchanged for the stress range under consideration. On the basis of elastic behaviour a maximum increase in the unit weight of the material comprising the particles would be approximately 0.2–0.3 % at an effective stress of 62.1 MN m^{-2} and with $u = 0$ (from equation (9) of Bishop 1973). This increase is not significant in the present context. However, Rieke & Chilingarian (1974) have presented data showing substantial decreases in the unit weight of clay particles with increase in consolidation pressure, the values for illite, for example, being 2.68, 2.53 and 2.38 gf/cm^3 at consolidation pressures of 13.8, 98.5 and 243 MN m^{-2} respectively.

The description of the method by which these results were obtained is given by Cebell & Chilingarian (1972), who carried out a series of high pressure consolidation tests in an oedometer somewhat similar to that of Westman (1932). However, Cebell & Chilingarian assume that on undrained removal of the total stress the pore water continues to fill the void space whatever the magnitude of the consolidation pressure used. In the light of the test results described in the present paper this appears to the authors to be an untenable assumption.

If equation (23) is expressed as a relation between the dry unit weight of the unconfined soil sample and $G\gamma_w$, the unit weight of the clay particles, we have:

$$\frac{1}{\gamma_d} = \frac{1}{G\gamma_w} + \frac{w}{S\gamma_w}. \tag{A 2.2}$$

The value of $G\gamma_w$ can only be determined from measurements of γ_d and w if assumptions are made about the values of S and γ_w. The assumption that $S = 1.000$ (full saturation) and $\gamma_w = 1.000$ gf/ml leads to values of $G\gamma_w$ of 2.59 for London Clay and 2.22 for Kaolin consolidated at pressures of 62.1 and 6.9 MN m^{-2} respectively. Alternatively if the values of $G\gamma_w$ measured at low pressure of 2.81 and 2.64 gf/cm^3 are assumed to apply, the term $S\gamma_w$ has the value of 0.78 for London Clay and 0.73 for Kaolin. While the value of γ_w for the 9.8 % of water in the London Clay and the

19.0 % of water in the Kaolin is a matter of controversy, it is unlikely to differ from that of free water by as much as 22 and 27 % respectively. The assumption of partial saturation is more readily acceptable, though clearly a closer investigation of the factors influencing the values γ_w and G in clays subject to large changes in effective stress, pore pressure and void ratio would be of interest.

References

Aitchison, G. D. 1960 Relationships of moisture stress and effective stress functions in unsaturated soils. *Proc. Conf. Pore Pressure and Suction in Soils*, 47–52. London: Butterworth.

Agarwal, K. B. 1967 The influence of size and orientation of samples on the undrained strength of London Clay. Ph.D. Thesis, University of London.

Bishop, A. W. 1947 Strength variations in London Clay. *Silicates Industriels* **13**, 109–113.

Bishop, A. W. 1952 The stability of earth dams. Ph.D. Thesis, University of London.

Bishop, A. W. 1953 Private communication to Dr A. S. Laughton (see Skempton 1960).

Bishop A. W. 1958 Test requirements for measuring the coefficient of earth pressure at rest. *Proc. Brussels Conf. on Earth Pressure Problems* **1**, 2–14.

Bishop, A. W. 1959 The principle of effective stress. *Teknisk Ukeblad* no. 39, 859–863.

Bishop, A. W. 1960 The measurement of pore pressure in the triaxial test. *Proc. Conf. Pore Pressure and Suction Soils*, 38–46. London: Butterworth.

Bishop, A. W. 1966 Soils and soft rocks as engineering materials. *Inaug. Lect. Imp. Coll. Sci. Technol.* **6**, 289–313.

Bishop, A. W. 1971 Shear strength parameters for undisturbed and remoulded soil specimens. In *Stress–strain behaviour of soils*, 3–58, 134–139. Cambridge: Proc. Roscoe Memorial Symp.

Bishop, A. W. 1973 The influence of an undrained change in stress on the pore pressure in porous media of low compressibility. *Geotechnique* **23**, 435–442.

Bishop, A. W., Alpan, I., Blight, G. E. & Donald, I. B. 1960 Factors controlling the strength of partly saturated cohesive soils. *Proc. Res. Conf. on Shear Strength of Cohesive Soils*, Boulder, 503–532.

Bishop, A. W. & Bjerrum, L. 1960 The relevance of the triaxial test to the solution of stability problem *Proc. Res. Conf. Shear Strength of Cohesive Soils*, Boulder, 437–501.

Bishop, A. W. & Blight, G. E. 1963 Some aspects of effective stress in saturated and partly saturated soils. *Geotechnique* **13**, 177–197.

Bishop, A. W. & Eldin, G. 1950 Undrained triaxial tests on saturated sands and their significance in the general theory of shear strength. *Geotechnique* **2**, 13–32.

Bishop, A. W. & Henkel, D. J. 1953 Pore pressure changes during shear in two undisturbed clays. *Proc. 3rd Int. Conf. Soil. Mech., Zurich* **1**, 94–99.

Bishop, A. W. & Henkel, D. J. 1962 *The measurement of soil properties in the triaxial test*, 2nd ed. London: Edward Arnold.

Bishop, A. W., Webb, D. L. & Lewin, P. I. 1965 Undisturbed samples of London Clay from the Ashford Common Shaft: Strength–effective stress relation. *Geotechnique* **15**, 1–31.

Bishop, A. W., Webb, D. L. & Skinner, A. E. 1965 Triaxial tests on soil at elevated cell pressures. *Proc. 6th Int. Conf. Soil Mech., Montreal* **1**, 170–174.

Bjerrum, L. & Anderson, K. H. 1972 In-situ measurement of lateral pressures in clay. *Proc. 5th Europ. Conf. Soil Mech., Madrid* **1**, 11–20.

Blight, G. E. 1967 Horizontal stresses in stiff and fissured lacustrine clays. *4th Regional Conf. Soil. Mech., Cape Town.*

British Standards Institution 1967 *Methods of testing soils for civil engineering purposes*. B.S. 1377.

Bruhn, R. W. 1972 A study of the effects of pore pressure on the strength and deformability of Berea Sandstone in triaxial compression. *U.S. Department of the Army Technical Report – Engineering Study*, no. 52.

Cebell, W. A. & Chilingarian, G. V. 1972 Some data on compressibility and density anomalies in halloysite, hectorite and illite clays. *Am. Assoc. Petroleum Geol. Bull.* **56**, 796–821.

Croney, D. & Coleman, J. D. 1960 Pore pressure and suction in soil. *Proc. Conf. Pore Pressure and Suction in Soils*, 31–37. London: Butterworth.

Donald, I. B. 1961 The mechanical properties of saturated and partly saturated soils with special reference to the influence of negative pore water pressures. Ph.D. Thesis, University of London.

Gibson, R. E. 1953 Experimental determination of the true cohesion and true angle of internal friction in clays. *Proc. 3rd Int. Conf. Soil Mech., Zurich* **1**, 126–130.

Golder, H. Q. & Skempton, A. W. 1948 The angle of shearing resistance in cohesive soils for tests at constant water content. *Proc. 2nd Int. Conf. Soil Mech., Rotterdam* **1**, 185–192.

Green, R. B. 1951 Ordinary liquid water-substance, its thermodynamic properties, dynamic behaviour and tensile strength. D.Sc. thesis, Massachusetts Institute of Technology.

Holmes, J. W. 1955 Water sorption and swelling of clay blocks. *J. Soil Sci.* **6**, 200–208.

Hvorslev, M. J. 1937 Uber die Festigkeitseigenschaften gestörter bindiger Böden. *Ingvidensk. Skr. A*, No 45. (English translation no. 69–5, Waterways Experiment Station, Vicksburg, Miss., 1969.)

Jurgenson, L. 1934 The shearing resistance of soils. *J. Boston Soc. civ. Engrs.* **21**, 242–275.

Kumapley, N. K. 1969 Triaxial tests on clays and silts at elevated cell pressures. Ph.D. Thesis, University of London.

Ladd, C. C. & Lambe, T. W. 1963 Shear strength of saturated clays. *Symp. on Laboratory Shear Testing of Soils, Ottawa, ASTM STP* 361, 342–371.

Lambe, T. W. & Whitman, R. V. 1969 *Soil mechanics*. New York: John Wiley.

Rieke, H. H. & Chilingarian, G. V. 1974 *Developments in sedimentology*. London: Elsevier.

Schmertmann, J. H. & Osterberg, J. O. 1960 An experimental study of the development of cohesion and friction with axial strain in saturated cohesive soils. *Proc. Res. Conf. Shear Strength of Cohesive Soils*, Boulder, 643–694.

Simons, N. 1958 Discussion on 'Test requirements for measuring the coefficient of earth pressure at rest'. *Proc. Brussels Conf. on Earth Pressure Problems* **3**, 50–53.

Skempton, A. W. 1953 Soil mechanics in relation to geology. *Proc. Yorkshire geol. Soc.* **29**, no. 3.

Skempton, A. W. 1954 The pore pressure coefficients A and B. *Geotechnique* **4**, 143–147.

Skempton, A. W. 1960 Effective stress in soils, concrete and rocks. *Proc. Conf. Pore Pressure and Suction in Soils*, pp. 4–16. London: Butterworth.

Skempton, A. W. 1961 Horizontal stresses in an overconsolidated Eocene clay. *Proc. 5th Int. Conf. Soil Mech., Paris*, **1**, 351–357.

Skempton, A. W. & Sowa, V. A. 1963 The behaviour of saturated clays during sampling and testing. *Geotechnique* **13**, 269–290.

Skinner, A. W. 1975 The effect of high pore water pressures on the mechanical behaviour of sediments. Ph.D. Thesis, University of London.

Taylor, D. W. 1944 Cylindrical compression research program on stress-deformation and strength characteristics of soils. *M.I.T. 10th Progress Report to US Engineers Department*.

Taylor, D. W. 1948 *Fundamentals of soil mechanics*. New York: John Wiley.

Temperley, H. N. V. 1946 The behaviour of water under hydrostatic tension. II. *Proc. phys. Soc.* **58**, 436.

Temperley, R. N. V. & Chambers, LL.G. 1946 The behaviour of water under hydrostatic tension. I. *Proc. phys. Soc.* **58**, 420.

Terzaghi, K. 1932 Trägfahigkeit der Flachgründungen. *Int. Assoc. Bridge Struct. Eng., Prelim. Publ.*, 659–683.

Terzaghi, K. 1936 The shearing resistance of saturated soils and the angle between the planes of shear. *Proc. 1st Int. Conf. Soil Mech., Harvard*, **1**, 54–56.

Terzaghi, K. 1943 *Theoretical soil mechanics*. New York: John Wiley.

Westman, A. E. R. 1932 The effect of mechanical pressure on the imbibitional and drying properties of some ceramic clays, I, II. *J. Am. Ceramic Soc.* **15**, 552–563, and **16**, 256–264.

Wissa, A. E. Z. 1969 Pore pressure measurement in saturated stiff soils. *J. Soil Mech. Fdn. Div. Am. Soc. civ. Engrs.* **95**, SM4, 1063–1073.

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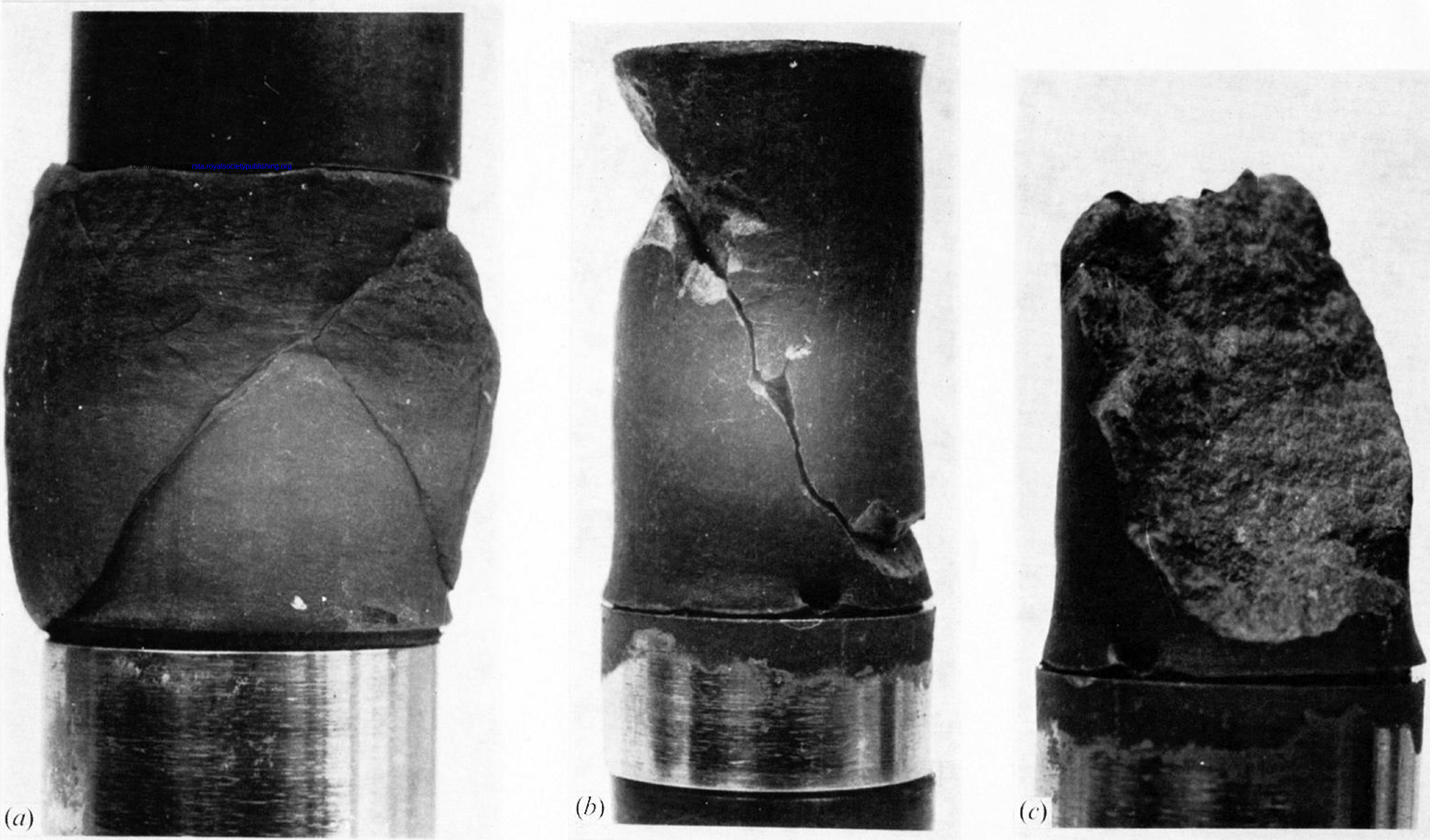

FIGURE 12. Samples consolidated at 62.1 MN m^{-2} (9000 lbf in^{-2}) and tested (*a*) with cell pressure equal to consolidation pressure (test no. 19 B) and (*b*) unconfined (test no. 21 B). Surface texture shown in (*c*) for unconfined test.